MINIMUM WEIGHT DESIGN OF CANTILEVER BRIDGES

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DEPARTMENT OF CIVIL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR
AUGUST 1971

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A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

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AUGUST 1971

CERTIFICATE

This is to certify that the thesis entitled 'MINIMUM WEIGHT DESIGN OF CANTILEVER BRIDGES' by P. KUMARAVEL SWAMY is a record of work carried out under my supervision and that it has not been submitted elsewhere for a degree.

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ACKNOWLEDGEMENTS

I would like to express my deep appreciation and gratitude to my thesis adviser, Dr. P. Dayaratnam, for initiating me to the work presented in this thesis. But for his sustained interest, guidance and encouragement to think originally, the work presented here would not have been possible.

I wish to express my appreciation to all my friends for their kind help. I am thankful to the staff of the Computer Centre, IIT/Kanpur for providing facilities on IBM 7044 system and to Mr. S.Kumar for the excellent typing.

P. KUMARAVEL SWAMI

SYNOPSIS

The cantilever construction has considerable relevance in long span bridge construction. The limiting spans of the cantilever and the effects of other parameters in the limiting spans were discussed in the first part. The spacing of the piers could be increased to a considerable extent by adopting the cantilever construction. The main part of the investigation is on the economic design criteria of the cantilever bridge. The weight of the individual segments and the relative weights of the individual segments with respect to the support segment have considerable influence on the requirement of the prestressing force and the total economy. The variation of the weight of the bridge and the prestressing steel requirements based on different variables in the bridge design are studied using a computer programme of nondimensionalized parameters. The work establishes a basis of selection of various quantities of design such as the depth, variation of the depth along the length of the beam, the shape factor and the relative thicknesses of the various portions of the cross section. The area of steel requirement is very sensitive to the weight of the segments. The ranges of economic percentage of steel for different range of spans are suggested. An illustrative example of a bridge using the results of the investigation is presented. The example illustrates that some assumptions which may not sound of much consequences, affect the design considerably.

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LIST OF SYMBOLS

α = self weight bending moment coefficient ($M_g/\gamma L^2$

β = ratio of area at the free end to that at support

γ = density of concrete

Δ = shape factor (y_b/y_t)

ϵ = eccentricity ratio (e/h)

ϵ_{req} = eccentricity required

ϵ_{limit} = eccentricity limit

n = P_e/P_t

θ = slope of the tendon

ρ = efficiency factor (r^2/h^2)

σ_{cu} = cube strength of concrete

σ_s' = ultimate tensile stress of prestressing steel

ω = depth factor ($h\,\sigma_{cu}/\gamma L^2$)

A = area of cross section of concrete

C.G.C. = centre of gravity of gross section

C.G.S. = centre of gravity of prestressing steel

I = moment of inertia

L = length of span

M = bending moment

M_a = moment due to external loads ($M_l + M_s$)

M_g = bending moment due to self weight

M_l = bending moment due to live load

M_s = bending moment due to superimposed load

M_t = bending moment due to total load ($M_a + M_s$)

M_u = ultimate bending moment

N_s = number of segments

P_e = effective prestress force after loss of prestress

P_t = prestressing force at transfer

$\bar{P}_{ti}$ = total horizontal component of prestressing force of i^{th} segment at transfer

P_{ti} = anchoraged prestressing force at i^{th} segment

R = moment ratio (M_a/M_g)

a = length of a segment

b = length of n segments

b_b = breadth of section at bottom

b_t = bredth of section at top

c_a = $A/b_t h$

c_{ce} = compression stress at effective load

c_{ct} = compression stress at transfer

c_f = b_b/b_t

c_i = I/bh^3

c_t = t_t/h

c_{te} = tension stress coefficient at effective load

c_{tt} = tension stress coefficient at transfer

c_w = b_w/b_t

c_{yb} = y_b/h

c_{yt} = y_t/h

c_{cc}', c_{ct}',
c_{te}', = limiting stress coefficients
c_{tt}'

d' = distance from C.G.S. to bottom fibre

e = eccentricity

e_i = distance of centroid of anchored steel at i^{th} segment from top fibre

$\bar{e}_i$ = net eccentricity of $\bar{P}_{ti}$ from C.G.C.

h = overall depth of beam

m = reinforcement ratio (= $P_t/A\sigma_{cu}$)

m_i = steel ratio of anchored steel at i^{th} segment

$\bar{m}_i$ = total steel ratio at i^{th} segment

r = radius of gyration

y_b = distance of bottom fibre from C.G.C.

y_t = distance of top fibre from C.G.C.

CHAPTER I

INTRODUCTION

1.1 Long Span Prestressed Bridges:

The practice of prestressing, although simple, has caused a revolution in the design of concrete structures. This method 'prestressing' is more interesting in the case of long span bridges with the choice of segmental construction. About 35 to 40 m spans were considered to be economical and satisfactory in reinforced concrete bridges (1)*. A good application of the high concrete strength in the design of concrete members and the skill developed for 30 years in actual field work have tended to give further confidence to the designer and builder. This has enlarged the scope of application of prestressing to long span bridges.

The Western Avenue Extension is an example of the role of prestressed concrete in intensive urban development (2). With a length of 4 KM of prestressed concrete construction, this is the longest elevated highway in Europe. The segmental units 28.6 m wide and weighing upto 135 tons are some of the biggest segments cast in U.K. A stretch of 1200 m of this roadway has been constructed using the segmental construction techniques. There are numerous simply supported solutions in the long span bridges, in the range of 30 to 45 m. The longest simply supported span is Mayo Oulo bridge in Cameroons with a span of 83.8 m (1). The maximum simply supported span reported in India is 52.5 m. The limitations of the launching equipment and the skill required to construct longer spans, with girder depth exceeding 3.7 m have acted as constraints. The solution has taken one of the following types of bridges (3).

---

* Numbers within the brackets indicate the references.

a. Balanced cantilever bridges (Fig.1.1) -
Spans 40 m to 85 m

b. Hammer head type (Fig. 1.2) -
Spans 30 m to 75 m

c. Cantilever type (Fig. 1.4 to Fig. 1.8) -
Spans 60 m to 180 m

d. Bridle-chord or Suspension bridge (4) (Fig.1.3) -
Spans 120 m to 300 m

a. Balanced Cantilever Type:

Balanced Cantilever bridges consist of spans simply supported over cantilevers as shown in Fig. 1.1. The problem in Balanced Cantilever bridge is not concreting and stressing of new units but of the fact that the central unit weighing heavily has to be supported on a heavy sturdy staging and transported. The number and type are too many, some of which cause high frictional forces on account of reverse curvature.

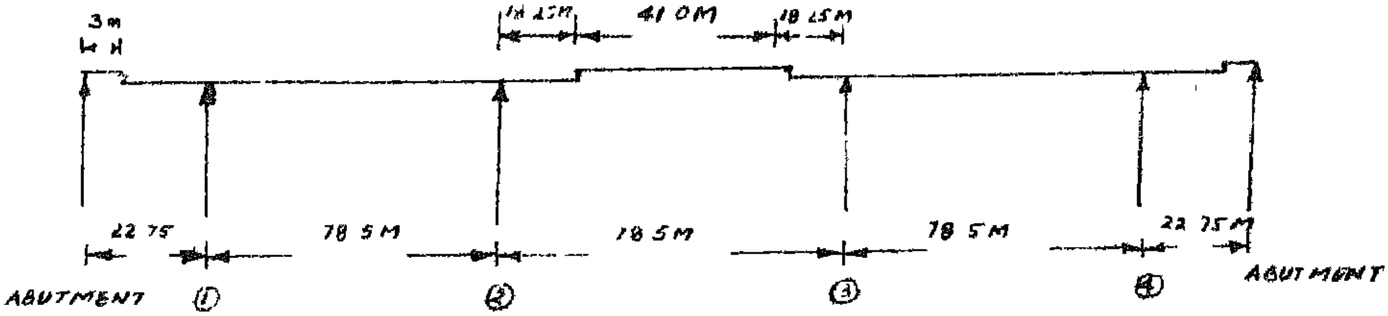
b. Hammer Head Type:

In Hammer head type (Fig.1.2) semicontinuity is realized as in the case of balanced cantilever but in a more elegant manner. It is a simple solution for the practical difficulties of cantilever construction. In this the maximum cantilever that can be provided for the hammer head is about 10 m and this means the maximum span that can be realized is approximately 75 m (1). The question of prestressing the Hammer head has often been considered but it has been found that it would be cheaper to go in for reinforced concrete.

A good example for Balanced Cantilever type is the completed Bhagirathi bridge (Fig. 1.1) and the Hammer head type is that of Kosi bridge (Fig. 1.2). The maximum span constructed in India to date by Hammer head system is 64.5 m.

c. Cantilever Type:

The cantilever type construction came into being as a natural extension of the philosophy of the Hammer head type where the cantilever extended beyond the perimeter of the foundation. This type of construction has been found to be convenient and competitive where the conditions of bridge site prohibit erection of centring in the river bed. Such a situation is found in creek areas where deep water and high tidal conditions are present. Also in deep ravines where



BHAGIRATHI BRIDGE

FIG 11 BALANCED CANTILEVER BRIDGE

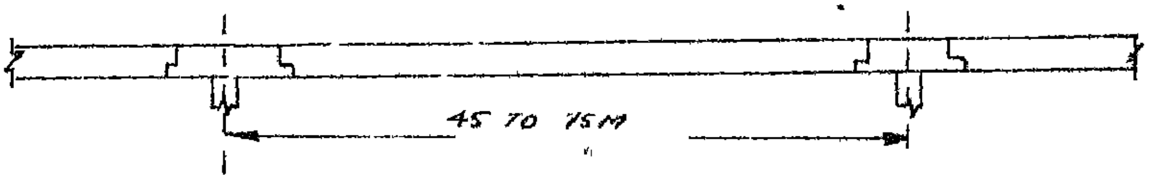
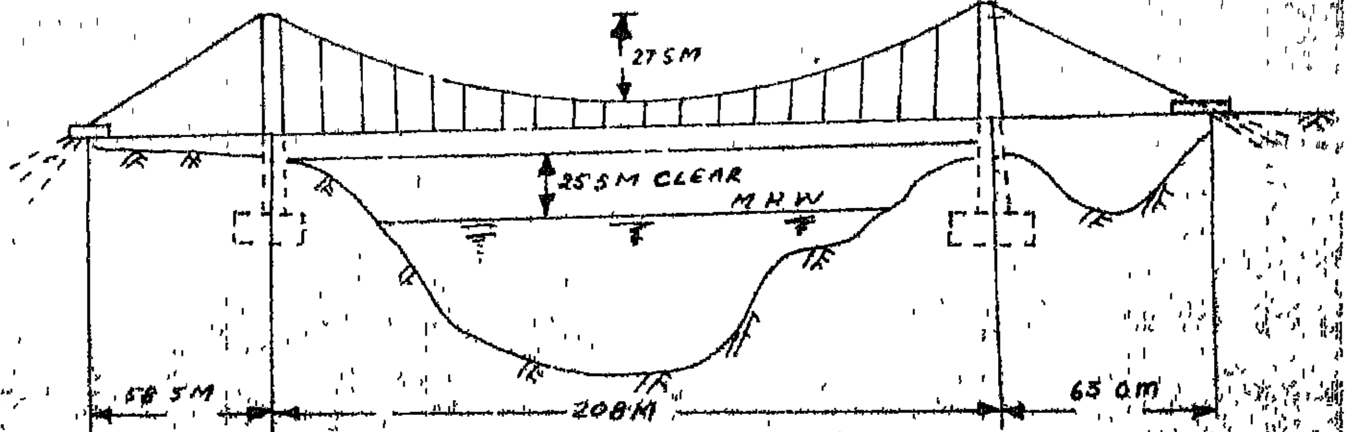


FIG 12- HAMMER HEAD SYSTEM



HUDSON HOPE BRIDGE

FIG 13 SUSPENSION BRIDGE

sudden fast flowing mountain streams are to be feared and in rivers with continuous navigation. The free cantilever method of construction with moving form is advantageous in areas with difficult accessibility. The method also provides a solution to the problem of building elevated highways in cities. One of the important advantages of the cantilever construction is the facility it provides for providing cable trajectories without reverse profile.

1.2 Cantilever Construction:

Basically, cantilever construction is a method of progressive construction of adding a series of precast symmetrical fashion on both sides of a supporting pier. As the cantilever is increased in length they balance each other. And each pier of castings added is prestressed by two or more cables passing over the pier. Sometimes on account of bed conditions, the shore cantilever may have to be supported on the abutment or may have to be shortened as compared to the river side cantilever, in which case, it may require provision of dead weight with non-structural concrete or sand. In mountain areas, shore side being of short length, the support is pinned to the rock or effectively counterbalanced by a counterweight block.

The type of joints to be adopted, where the two cantilevers from adjacent piers meet at about the centre of the span is entirely dependent on foundation strata. The junctions can (1 and 5) be classified as:

(i) Cantilevers with short suspended span, determinate for dead and live load (Fig. 1.4).

(ii) Cantilevers with a central hinge (Fig. 1.5).

(iii) Cantilevers with a short suspended span, made continuous for live load.

(iv) Cantilevers joined at centre of span (Fig. 1.6).

The general approach is to provide a hinge at the centre of span of cantilever, where the expected settlement of the foundation is within 25 percent of the deflection caused by liveload at the centre of span. Where the settlement is large, a short suspended span is normally proposed as in the case of Barack bridge at Badarpur (Fig. 1.4). Cantilevers joined at the centre of span, where foundations are resting on rock as at Bassein Creek bridge (Fig. 1.6), continuity is likely to give better riding qualities for traffice over the bridge.

The longest span in cantilever construction is reported under construction at the Parana Lake bridge, Brazilla (6). The structure has a total length of 440 m, a width of 13.50 m. The central span of 220 m and two end spans of 110 m will be constructed by cantilever method with cast

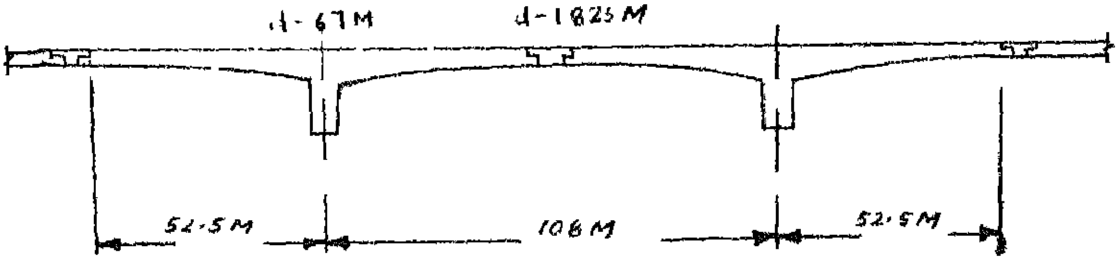


FIG 14 - BARAK BRIDGE AT BADARPUR

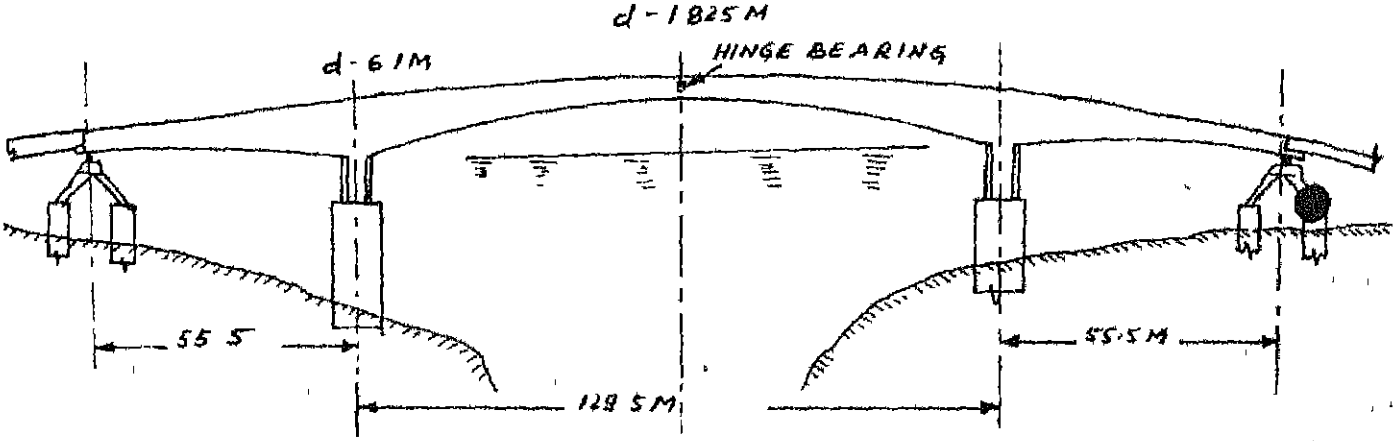


FIG 15 - BARAK BRIDGE AT SILCHAR (ASSAM)

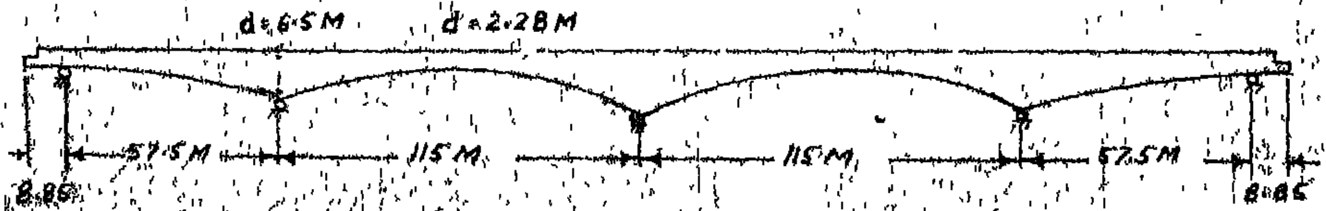
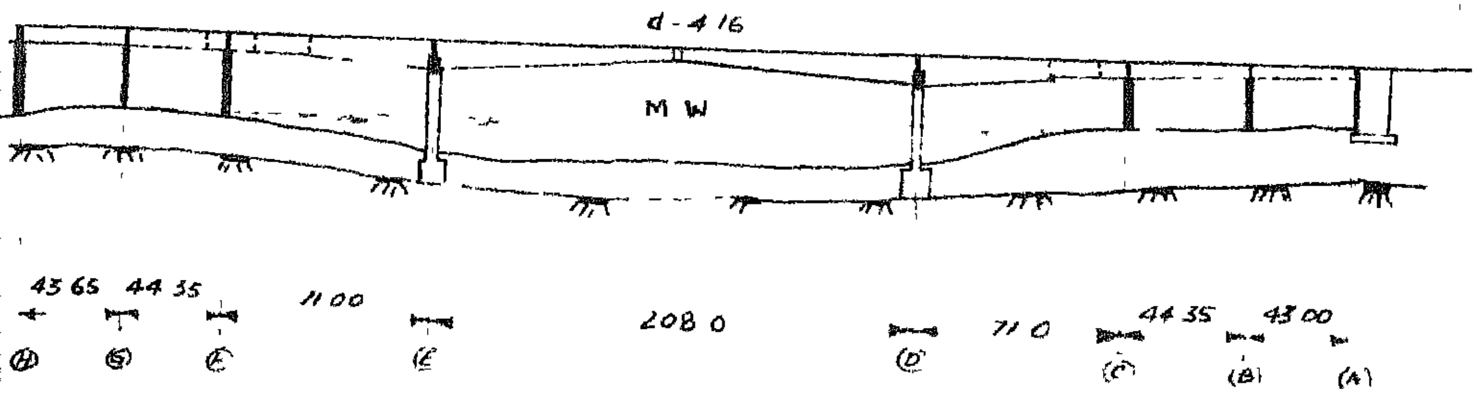
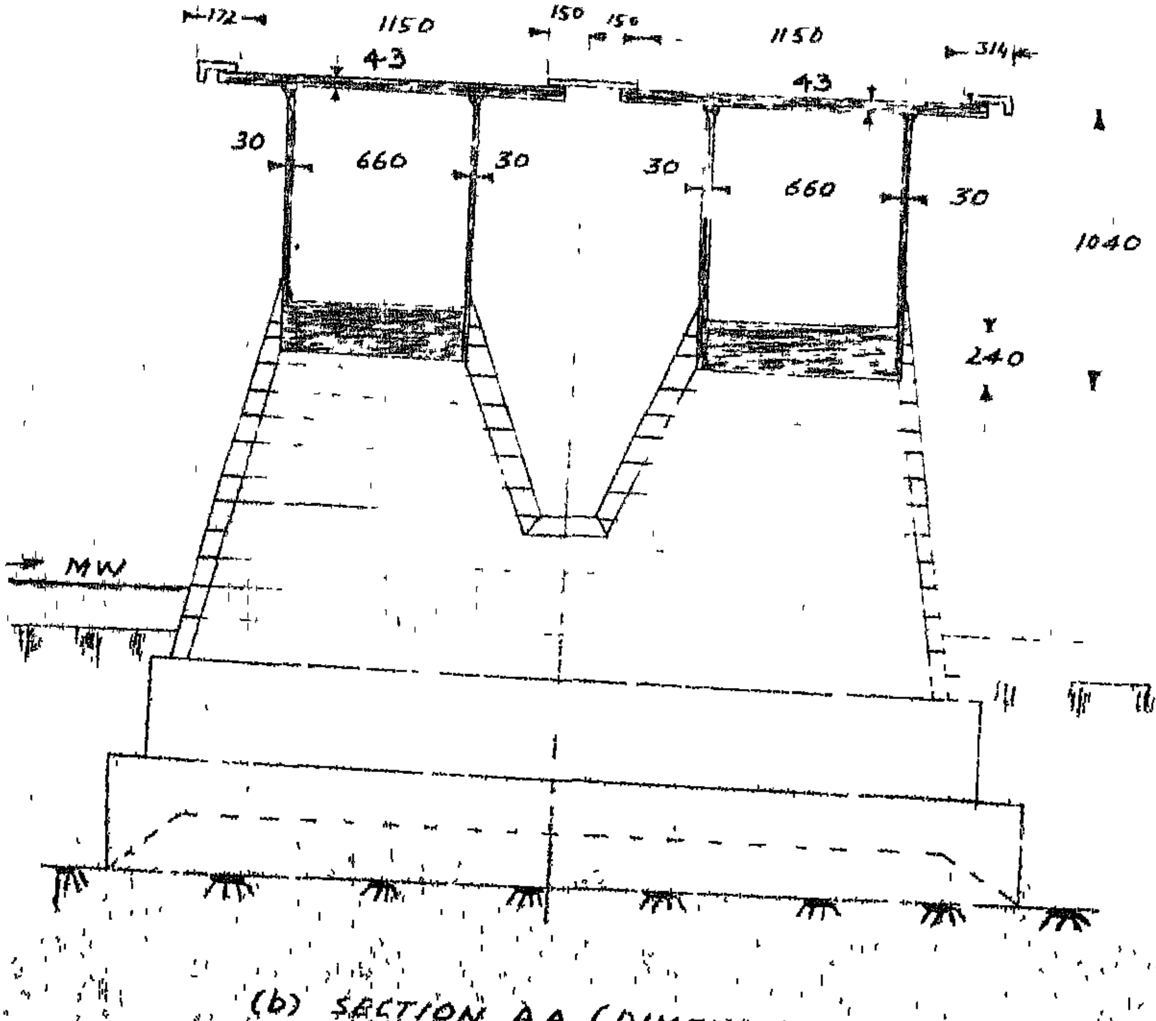


FIG. 1.6 - BASSEIN CREEK BRIDGE

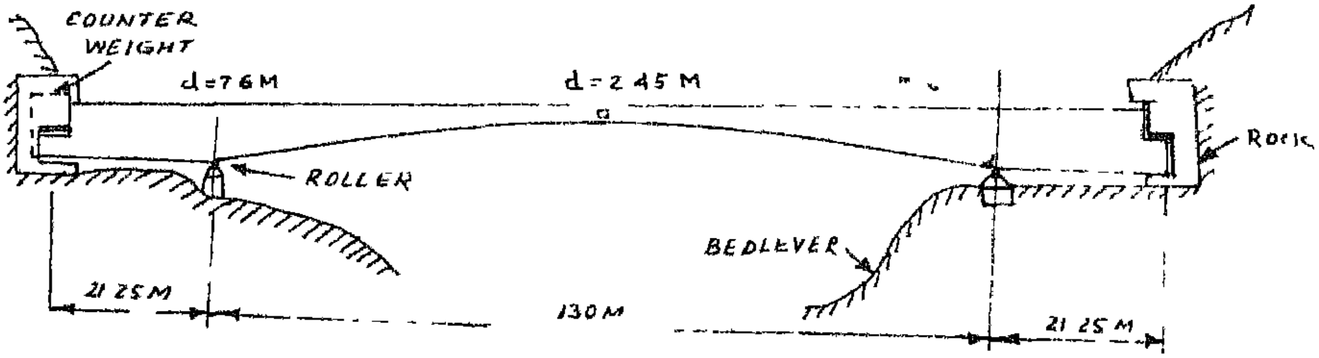


(a) LONGITUDINAL SECTION (DIMENSIONS IN METRE)

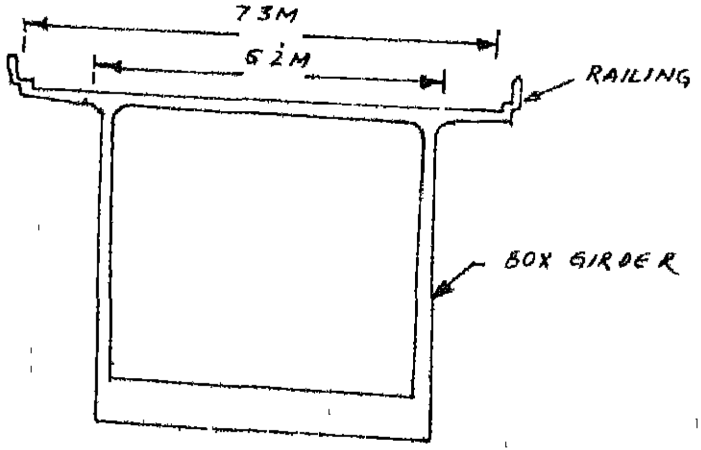


(b) SECTION AA (DIMENSIONS IN CM)

FIG 7.7 — BENDORF BRIDGE



ELEVATION



TYPICAL CROSS SECTION

FIG. -18- LUBHA BRIDGE

in situ sections of 5 m. The longest bridge span that has been constructed so far by cantilever method is the main span Bendorf Bridge (Fig. 1.7) over the Rhine with 208 m. The longest span of cantilever bridge in India is that of the Lubha bridge in Assam with a span of 140 m (Fig.1.8).

The cross sections of all the bridges described above are either of the 'T' or the box type. A box section on account of its high torsional rigidity appears to be most satisfactory for cantilever construction.

1.3 Statement of the Problem:

The cantilever construction is economical for long span bridges under special recommended cases. Precasting and launching have become a distinct possibility and spans from 90 to 140 m have been constructed in India by cantilever construction. Spans of 180 - 240 m are under study (3). Naturally the question arises, are there any limits on the span in the cantilever construction, and if at all any limit is there, then what are the bounds. In achieving this limit a rational basis of economic design method is developed.

This thesis presents the design of long span cantilever bridges based on minimum weight principles using the non-dimensionalized parameters. Khachaturian (9) has developed non-dimentionalized governing equations for the design of simply supported prestressed concrete beams based on allowable stresses. The equations are extended to cantilevers. The solutions of the equations are used in the computer programme to give complete design details of a two lane bridge with the IRC loading for different spans. The effects of the depth of beam, shape of the beam, cross sectional area variations and other parameters like live load to dead load ratio are illustrated with respect to various spans. In the present work, the minimum weight design of cantilever type bridge is studied with the permissible stress coefficients both at the transfer and service load conditions as the main design variable.

While optimising a prestressed concrete flexural member, the two factors which deserve considerations are the minimum weight and the minimum cost. The weight of the member influences the launching operations and the loads acting on the substructure. This minimum weight design is likely to bring savings in other items of construction and this is particularly true in the case of the cantilever construction. Economy can be achieved by minimizing the total cost of the member. The fluctuations in prices and many other cost parameters make the study of optimisation based on cost criteria of temporary significance. It was, hence thought logical to lay a design procedure for obtaining minimum weight design. A minimum weight section in which the stresses at transfer and in service satisfy the permissible stresses exactly (called as efficient section) is also likely to be minimum cost section provided the section does not demand extra steel from the ultimate flexural considerations.

In a fully stressed design of a prestressed concrete structure the stresses at transfer and working conditions are designed to the maximum permissible stresses. Because of the bounds imposed in long span bridges on depth/span ratio, top flange width/depth ratio etc., it is not always possible to have a full stress design. For various spans the range of maximum value of stresses are worked out for minimum weight design.

A procedure is also laid out for the effective curtailment, the eccentricity of steel in the intermediate sections and the cross sectional and height variations along the span. A complete numerical example is worked out for a cantilever of span 90 m in Chapter III. Results of the numerical example and the conclusions drawn from the investigations are also discussed in Chapter III.

CHAPTER II

FORMULATION OF THE DESIGN PROBLEM

2.1 Governing Equations for Working Load Design:

In working load design, the stresses in member due to various working loads are calculated assuming elastic behaviour of the material and these stresses are kept below certain allowable stresses. The stresses are adopted based on an arbitrary safety of the material.

Prestressed concrete beams are subjected to two limiting conditions of loading,

(i) Loads at transfer

(ii) Critical working loads.

At transfer condition, prestressing force is maximum and concrete strength is minimum. At working load condition, concrete strength is maximum and prestressing force is minimum while superimposed and live loads are at maximum.

Extreme stresses occur at bottom and top fibres of a beam at a given cross section for each of the load condition- Hence the working load design is governed by the extreme fibre stresses in the two load conditions. The expressions for the stresses of a cantilever beam (Fig. 2.1) are

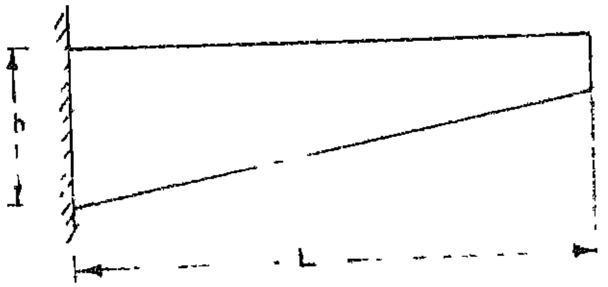


FIG 2.1 - CANTILEVER BEAM

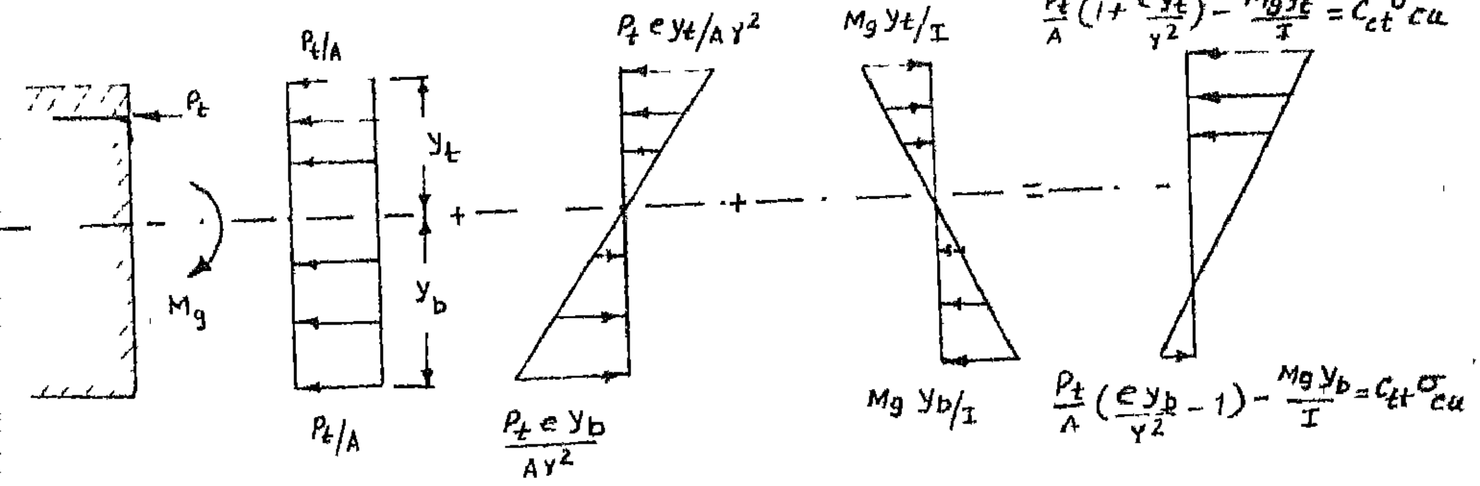

P_t
M_g
P_t/A
P_t/A
y_t
y_b
+
$P_t e y_t/A\gamma^2$
$\frac{P_t e y_b}{A\gamma^2}$
+
$M_g y_t/I$
$M_g y_b/I$
=
$\frac{P_t}{A}(1+\frac{e y_t}{\gamma^2}) - \frac{M_g y_t}{I} = C_{ct}\sigma_{cu}$
$\frac{P_t}{A}(\frac{e y_b}{\gamma^2}-1) - \frac{M_g y_b}{I} = C_{tt}\sigma_{cu}$

FIG 2.2 - STRESS CONDITION AT TRANSFER

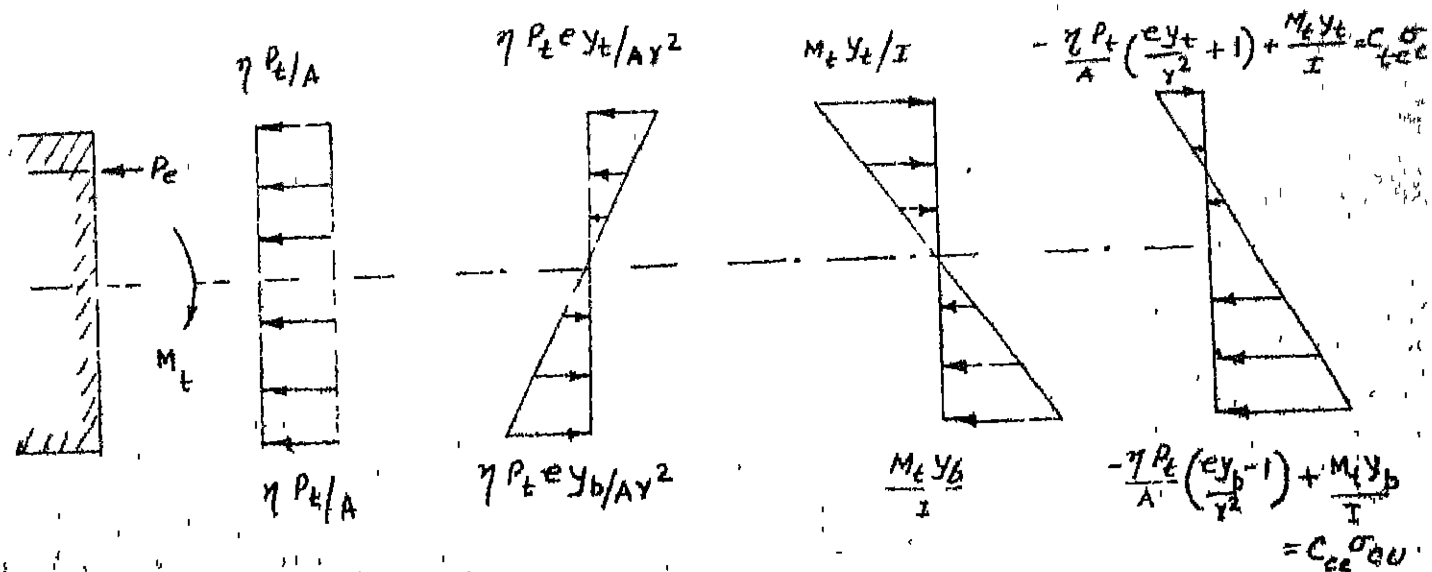

P_e
M_t
$\eta P_t/A$
$\eta P_t/A$
$\eta P_t e y_t/A\gamma^2$
$\eta P_t e y_b/A\gamma^2$
$M_t y_t/I$
$\frac{M_t y_b}{I}$
$-\frac{\eta P_t}{A}(\frac{e y_t}{\gamma^2}+1) + \frac{M_t y_t}{I} = C_{te}\sigma_{ce}$
$-\frac{\eta P_t}{A}(\frac{e y_b}{\gamma^2}-1) + \frac{M_t y_b}{I} = C_{ce}\sigma_{cu}$

FIG 2.3 - STRESS CONDITIONS AT WORKING LOAD

(i) Stress condition at transfer (Fig. 2.2) :

$$\frac{P_t}{A}\left(\frac{ey_t}{r^2} + 1\right) - \frac{M_g\, y_t}{I} = c_{ct}\,\sigma_{cu} \leq c'_{ct}\,\sigma_{cu} \qquad (2.1)$$

$$\frac{P_t}{A}\left(\frac{ey_b}{r^2} - 1\right) - \frac{M_g\, y_b}{I} = c_{tt}\,\sigma_{cu} \leq c'_{tt}\,\sigma_{cu} \qquad (2.2)$$

(ii) Stress condition at working (Fig. 2.3) :

$$\frac{-\eta\, P_t}{A}\left(\frac{e\, y_b}{r^2} - 1\right) + \frac{M_t\, y_b}{I} = c_{cc}\,\sigma_{cu} \leq c'_{cc}\,\sigma_{cu} \qquad (2.3$$

$$\frac{-\eta\, P_t}{A}\left(\frac{e\, y_t}{r^2} + 1\right) + \frac{M_t\, y_t}{I} = c_{te}\,\sigma_{cu} \leq c'_{te}\,\sigma_{cu} \qquad (2.4)$$

where

P_t = prestressing force at transfer

A = gross cross sectional area of the beam

I = moment of inertia of the beam

r = radius of gyration of the beam

M_g = bending moment due to self weight of the beam

M_t = bending moment at working load ($M_t = M_g+M_s+M_l$)

M_s = bending moment due to superimposed load

M_l = bending moment due to live load

M_a = $M_s + M_l$

σ_{cu} = ultimate cube strength of concrete

η = effectiveness of prestressing force (P_e/P_t)

c_{cc} = compression stress coefficient at working load

c_{ct} = compression stress coefficient at transfer

c_{te} = tension stress coefficient at effective load

c_{tt} = tension stress coefficient at transfer

c'_{cc}, c'_{ct}, c'_{te},

c'_{tt} = limiting allowable stress coefficients

y_b = distance of bottom fibre from C.G.C.

y_t = distance of top fibre from C.G.C.

The above four equations can be simplified by introducing non-dimensional parameters. The equations for allowable stress condition in non-dimensionalized parameters expressed as suggested by Khachaturian (2) are

$$m\left[\frac{\epsilon\Delta}{(1+\Delta)} - 1\right] - \frac{\Delta}{\alpha\omega\rho(1+\Delta)} = c_{tt} \leq c'_{tt} \tag{2.5}$$

$$m\left[\frac{\epsilon}{(1+\Delta)} + 1\right] - \frac{1}{\alpha\omega\rho(1+\Delta)} = c_{ct} \leq c'_{ct} \tag{2.6}$$

$$-\eta\, m\left[\frac{\epsilon\Delta}{(1+\Delta)} - 1\right] + \frac{(1+R)\Delta}{\alpha\omega\rho(1+\Delta)} = c_{cc} \leq c'_{ce} \tag{2.7}$$

$$-\eta\, m\left[\frac{\epsilon}{(1+\Delta)} + 1\right] + \frac{(1+R)}{\alpha\omega\rho(1+\Delta)} = c_{te} \leq c'_{te} \tag{2.8}$$

where

γ = density of concrete

h = depth of the beam

L = cantilever span

Steel properties:

(i) $\frac{P_t}{A\,\sigma_{cu}} = n$ ... (reinforcement ratio)

(ii) $\frac{e}{h} = \epsilon$ ... (eccentricity factor)

Section properties:

(iii) $\frac{h\,\sigma_{cu}}{\gamma\,L^2} = \omega$ ... (depth factor)

(iv) $(\gamma/h)^2 = \rho$ ... (efficiency factor)

(v) $(y_b/y_t) = \Delta$ ... (shape factor)

(vi) $(M_a/M_g) = R$ ... (moment ratio)

(vii) $\alpha = (\gamma A\,L^2/M_g)$ ...(bending moment coefficient)

2.2. Solutions of the Governing Equations:

Known and unknown quantities of the above four equations 2.5 to 2.8 may be classified as

c_{ct}, c_{tt}, c_{ce}, c_{te} (known or atleast assumed within the limit

$\eta, \omega, \rho, \Delta$, R, α, n and ϵ (unknown)

Out of the eight unknowns, η can be reasonably estimated and recomputed if necessary. With this assumption there are seven unknowns in the four equations. α is dependent on R and section properties. Hence the unknowns are six in the four equations which means that atleast two of the non-dimensionalized parameters have to be estimated to get a unique solution. Arbitrary estimate of any two parameters is to be done on practical and economic considerations.

(a) Depth factor ($\omega = \dfrac{h\,\sigma_{cu}}{\gamma\, L^2}$) :

The expression 'ω' contains three known quantities (σ_{cu}, γ, L) and unknown quantity (h). Depth of the beam in many cases is controlled by architectural, deflection and site conditions. For cantilever bridges, it can reasonably be assumed as $\frac{L}{10}$ to $\frac{L}{15}$. The effect of depth of the beam on economy is illustrated (Chapter III).

(b) Efficiency Ratio ($\rho = r^2/h^2$) :

For a given depth the flexural efficiency of the section increases with the increase in radius of gyration. Higher the value of ρ, higher is the efficiency of the section. A slender web increases the efficiency ratio. The effect and mode of selection of efficiency ratio for economy was discussed in (8).

(c) Shape factor ($\Delta = y_b/y_t$) :

For symmetric I sections (any symmetric sections), the value of shape factor is unity. For heavy live loads and for long cantilever spans, the bottom flange has to be heavier in which case Δ should be chosen between 0.60 and 0.9. Even though the range of Δ is small, the effect on the limiting equations is considerable, so it is desirable that this value is not assumed to start within the design.

(d) Moment Ratio ($R = M_a/M_g = M_s+M_l/M_g$)

It is difficult to fix any limit to the range of the value of R. As the length of span increases the ratio R decreases. The value of R varies from 6.0 (for short spans) to 0.5 (for long spans). It should not be assumed but should be solved from equations.

(e) Reinforcement Ratio : ($m = P_t/A\,\sigma_{cu}$)

P_t/A may be defined as average stress. The reinfrocement ratio is simply the ratio of average stress to the strength of concrete. For all practical purposes, it varies from 0.15 to 0.40. The value of m should be computed from the governing equations for economical section.

(f) Eccentricity Ratio : ($\epsilon = e/h$)

Eccentricity ratio is a measure of effective utilization of prestressing force. As the value of ϵ increases, the required prestressing force decreases. ϵ can be taken as high as possible but it has to be bounded by practical considerations such as cover, transfer condition etc. The limits of eccentricity ratio vary from 0.25 to 0.60. The extreme value of ϵ is limited by cover distance. If d' is the minimum cover required then,

$$\epsilon_{limit} = \frac{y_t - d'}{h} = \frac{y_t}{y_t + y_b} - \frac{d'}{h} = \frac{1}{1+\Delta} - \frac{d'}{h} \qquad (2.9)$$

Solution of the Equations:

In many practical problems, values of and ρ may be assumed and then the four unknowns (Δ, m, ϵ, and R) with the four equations are left which can be solved exactly for a given set of values of c's. The four equations (2.5 to 2.8) are rearranged below.

Multiply Eq. (2.5) by η and add to Eq.(2.7)

$$\frac{\Delta\,(1+R-\eta)}{\alpha\omega\rho\,(1+\Delta)} = \eta\, c_{tt} + c_{ce} \qquad (2.10)$$

Multiply Eq. (2.6) by η and add to Eq. (2.8)

$$\frac{(1+R-\eta)}{\alpha\omega\rho\,(1+\Delta)} = \eta\, c_{ct} + c_{te} \qquad (2.11)$$

Eq. (2.10) over Eq. (2.11) yields to

$$\Delta = \frac{\eta\, c_{tt} + c_{ce}}{\eta\, c_{ct} + c_{te}} \tag{2.12}$$

Addition of Eqs. (2.10) and (2.11) yields

$$\frac{1+R-\eta}{\alpha\omega\rho(1+\Delta)}\,(1+\Delta) = \eta\,(c_{ct} + c_{tt}) + (c_{te} + c_{ce})$$

or

$$R = \alpha\omega\rho\left[(c_{te} + c_{ce}) + \eta\,(c_{ct} + c_{tt})\right] - (1-\eta) \ldots (2.13)$$

Dividing Eq. (2.5) by Δ and subtracting from Eq. (2.6)

$$m = \frac{\Delta\, c_{ct} - c_{tt}}{(1+\Delta)} \tag{2.14}$$

Similarly from Eqs. (2.7) and (2.8)

$$m = \frac{c_{ce} - \Delta c_{te}}{\eta\,(1+\Delta)} \tag{2.15}$$

Substituting the value of Δ in Eq. (2.14)

$$m = \frac{c_{ct}\, c_{ce} - c_{tt}\, c_{te}}{\eta\,(c_{tt} + c_{ct}) + (c_{te} + c_{ce})} \tag{2.16}$$

Adding Eqs. (2.5) and (2.6)

$$\epsilon = \left[\rho\,(c_{tt} + c_{ct}) + \frac{1}{\alpha\omega}\right]\left[\frac{1}{m}\right] \tag{2.17}$$

$$\epsilon = \left[\rho(c_{tt}+c_{ct}) + \frac{1}{\alpha\omega}\right]\left[\frac{(c_{te}+c_{ce}) + \eta\,(c_{tt}+c_{ct})}{(c_{ct}\, c_{ce} - c_{tt}\, c_{ce})}\right] \tag{2.18}$$

Eqs. (2.12) and (2.13) define the properties of section and Eqs. (2.16) and (2.17) define the prestressing steel and its location. The above equations are used to get an economic design.

2.3 Limiting Span:

The limiting spans and the importance of the shape factor based on Indian Code of practice are discussed here for the case of simply supported span (10) and the cantilever span.

a. Simply Supported Spans:

The four governing equations for maximum stresses at the extreme fibres of a simply supported beam at transfer and working load conditions can be written in non-dimensionalized (11) parameters as

$$m \left[\frac{\epsilon}{(1+\Delta)} - 1 \right] - \frac{1}{\alpha \omega \rho (1+\Delta)} \leq c'_{tt} \qquad (2.19)$$

$$m \left[\frac{\epsilon \Delta}{(1+\Delta)} + 1 \right] - \frac{\Delta}{\alpha \omega \rho (1+\Delta)} \leq c'_{ct} \qquad (2.20)$$

$$-\eta\, m \left[\frac{\epsilon}{(1+\Delta)} - 1 \right] + \frac{1+R}{\alpha \omega \rho (1+\Delta)} \leq c'_{ce} \qquad (2.21)$$

$$-\eta\, m \left[\frac{\Delta \epsilon}{(1+\Delta)} + 1 \right] + \frac{(1+R)\Delta}{\alpha \omega \rho (1+\Delta)} \leq c'_{te} \qquad (2.22)$$

The solution of the Eqs. (2.19) to (2.22) for the equality sign gives

$$\Delta = \frac{\eta\, c'_{ct} + c'_{te}}{\eta\, c'_{tt} + c'_{ce}} \qquad (2.23)$$

From Eq.(2.23), it may be observed that once the permissible stresses are fixed, the shape of the section (y_b/y_t) is automatically fixed. If the maximum permissible stresses as per code are fully satisfied, the cross sectional shape is fixed and consequently the limits on the span. The Eqs.(2.19) and (2.20) fix the lower bound for the spans while Eqs. (2.21) and (2.22) fix the upper bound.

Using post tension, the maximum limit for concrete mix is M 530 as per I.S.Code and the corresponding permissible stress coefficients are

$c'_{tt} = c'_{te} = 0.019$ (corresponds to 10 kg./sq.cm. tension)

$c'_{ct} = 0.043 \qquad c'_{ce} = 0.31$

Substitution of these coefficients in Eq. (2.23) gives,

$$\Delta = 1.15$$

Permitting approximately 1.25 percent reinforcement, the non-dimensionalized factor is

$$n = \frac{P_t}{A\,\sigma_{cu}} = \frac{0.7\,\sigma'_s\,A_s}{A\,\sigma_{cu}} = \frac{0.7 \times 15000}{530}\left(\frac{A_s}{A}\right) \cong 0.25$$

where σ_s' = ultimate tensile strength of high tensile steel (used as 15000 kg./sq.cm.)

The efficiency ratio ρ for most I and box sections is in the range of 0.11 and 0.16 and the eccentricity is governed by the shape factor Δ.

$$\epsilon = \frac{e}{h} = \frac{y_b - d'}{h} = \frac{\Delta}{1+\Delta} - \frac{d'}{h} = \frac{1.15}{2.15} - 0.1 = 0.420$$

So using, $\rho = 0.14$

the Eqs. (2.19 to 2.22) respectively reduce to

$$L \geq 38.4\ (h/L)\ \alpha \qquad (2.24)$$

$$L \geq 130.0\ (h/L)\ \alpha \qquad (2.25)$$

$$L \leq \frac{270}{1+R}\ \left(\frac{h}{L}\right)\ \alpha \qquad (2.26)$$

$$L \leq \frac{350}{1+R}\ \left(\frac{h}{L}\right)\ \alpha \qquad (2.27)$$

For simply supported beams, $\alpha = 8$, let $h/L = 1/12$ and $R = 0.5$ (for a limiting case of infinite self weight $R = 0$), Eqs. (2.24) to (2.27) reduce to

$$\left.\begin{array}{l} L \geq 38.4 \text{ metres} \\ L \geq 86.2 \text{ metres} \\ L \leq 120.0 \text{ metres} \\ L \leq 160.0 \text{ metres} \end{array}\right. \qquad (2.28)$$

In the above calculations by changing the value of steel ratio (m) to 0.15 and $h/L = 1/15$ and the values of other parameters being kept the same, the limits work out to be

$$L \geq 14.4 \text{ metres}$$
$$L \leq 77.2 \text{ metres}$$

The limits fixed above are not the absolute limits but approximate limits based on maximum permissible stresses for M 530 concrete. It is possible to alter these limits by adopting some stresses that are permissible stresses. For example instead of permitting a tension stress of 10 kg./sq.cm. at transfer condition, a compressive stress $0.089\ \sigma'_{cu}$ is permitted while keeping the remaining stresses to the maximum permissible, i.e.

$$c_{tt} = -\ 0.089 \qquad c'_{te} = 0.019$$

$$c'_{ct} = 0.43 \qquad c'_{cc} = 0.31$$

The value of Δ, and ϵ work out as 1.68 and 0.530 respectively, so using $\rho = 0.14$ and $n = 0.25$ (same as previous case)

$$\begin{aligned} L &\geq 161\ (h/L)\ \alpha \\ L &\geq 206\ (h/L)\ \alpha \\ L &\leq \frac{334}{1+R}\left(\frac{h}{L}\right)\alpha \\ L &\leq \frac{369}{1+R}\left(\frac{h}{L}\right)\alpha \end{aligned} \qquad (2.29)$$

For simply supported beam of $h/L = 1/12$ and $R = 0.5$ the above limits work out to be

$$\begin{aligned} L &> 130 \text{ metres} \\ L &< 143 \text{ metres} \end{aligned} \qquad (2.30)$$

Similarly the limits could be varied by selecting appropriate working stresses.

(b) Cantilever Spans:

Using the stress coefficients for M 530 (same as in the simply supported spans)

$$c_{tt} = -\ 0.089 \qquad c'_{te} = 0.019$$

$$c'_{ct} = 0.43 \qquad c'_{ce} = 0.31$$

the shape factor and eccentricity ratio ϵ for the cantilever span are, (calculated from Eqs. (2.12) and (2.9))

$$\Delta = 0.595 \qquad \epsilon = 0.54$$

Keeping $\rho = 0.14, \qquad n = 0.25$

the Eqs. (2.5) to (2.8) respectively reduce to

$$\begin{aligned} L &\geq 167\ (h/L)\ \alpha \\ L &\geq 208\ (h/L)\ \alpha \\ L &\leq \frac{336}{1+R}\ \left(\frac{h}{L}\right)\ \alpha \\ L &\leq \frac{384}{1+R}\ \left(\frac{h}{L}\right)\ \alpha \end{aligned} \qquad (2.31)$$

For cantilever spans h/L can be as high as 1/10, making $\alpha = 4.75$ and let $R = 0.5$, the limit is

$$L < 106 \text{ metres} \tag{2.32}$$

Hence maximum limiting pier spacing in cantilever construction,

$$2\,L \cong 212 \text{ metres} \tag{2.33}$$

It will be seen from the results the span limit is high for cantilever construction. Moreover, the depth of simply supported beam has to be 12 m (L/12) for a span of 143 m and has to be uniform throughout the span. Whereas in cantilever construction of pier spacing of 212 m the depth is only10.6m (2L/20) at supports and can be reduced to 2L/50 at the free end of the span. The span limits are most sensitive to the permissible stress coefficients and the depth/ span ratio values. The span can also be extended by the increas in cube strength of concrete.

The above span limits are not absolute but relative. In long span bridges the limitation in depth/span ratio, minimum top flange thickness, top flange width in accordance with the traffic width impose certain amount of restriction on the shape of the section. Hence it is not always possible to obtain a fully stressed design, as seen from Eqs. (2.12) and (2.13). The limits for the long spans could be adjusted to some extent by the choice of the shape of the section while permitting stresses less than the maximum permissible stresses.

2.4 Stresses at Intermediate Sections:

In cantilever constructed bridge, each segment is progressively prestressed as the construction progresses. Unlike the simply supported bridge, the cantilever bridge has longitudinal variation in sectional area, height and prestressing steel. The proper combinations of these variations should be chosen so as to enable that all the sections of each segment are within the permissible stress and to effectively utilise the concrete and steel. In view of these the following set of equations are derived.

(a) Determination of n and ϵ :

Let the span of cantilever=L (Fig. 2.4) with number of segments = N_s. The cable profile usually adopted is shown in Fig. 2.4.

Hence length of each segment : $a = \frac{L}{N_s}$ (2.34)

The prestressing steel is made inclined to pass through 'n' segments.

From Fig. 2.4, $b = n \quad a$ (2.35)

where b = length of 'n' segments

Let A_0 and A_b are the cross sectional areas at free and support ends respectively.

Considering i^{th} segment starting from free end (Fig. 2.5), Let the total horizontal component of prestressing force at transfer $= \bar{P}_{ti}$

area of cross section $= A_i$

the prestressing force introduced at that section $= P_{ti}$

the inclination of the cable to horizontal corresponding to P_{ti} $= \theta_i$

distance of P_{ti} from top fibre $= e_i$

net eccentricity of total prestressing force $\bar{P}_{ti}$ $= \bar{e}_i$

shape factor of the section $= \Delta_i$

$$= \frac{\text{distance of bottom from C.G.C.}}{\text{distance of top from C.G.C.}} = \frac{y_{bi}}{y_{ti}}$$

Representing j as any one of intermediate sections between 1st and i^{th} segment and equating the total prestressing force (Fig. 2.5 and 2.6), neglecting the loss of prestress due to friction at kinks,

$$\bar{P}_{ti} = \sum_{j=1}^{i-n} P_{tj} + \sum_{j=i-n+1}^{i} P_{tj} \cos \theta_j \qquad (2.36)$$

Substituting,

$$P_{ti} = m_i \, A_i \, \sigma_{cu}$$
$$P_{tj} = m_j \, A_j \, \sigma_{cu} \qquad (2.37)$$

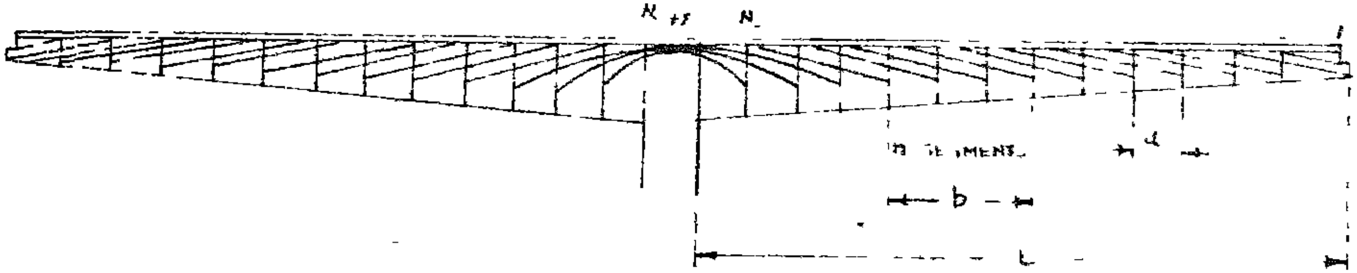
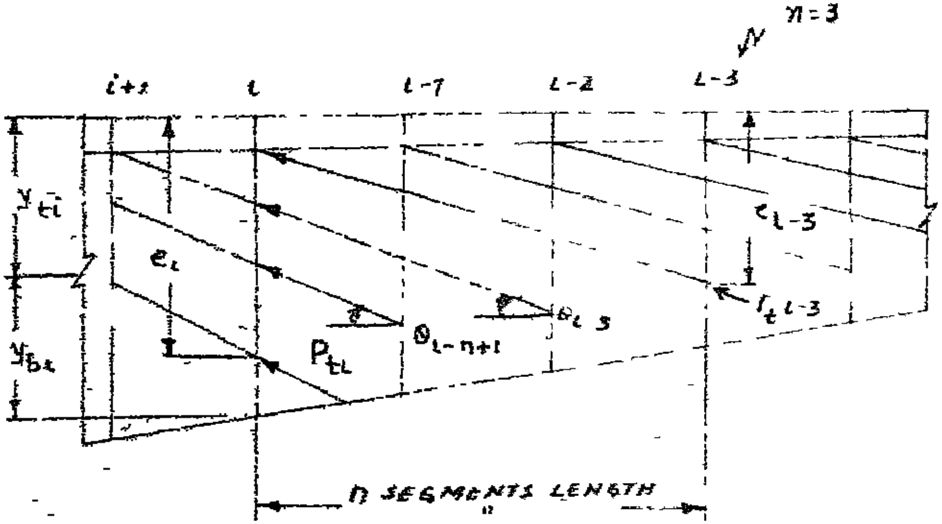


FIG 2.4 - ELEVATION - CABLE PROFILE



LOOPING DOWN = n SEGMENTS b

FIG. 2.5 - "i^{th}" SEGMENT

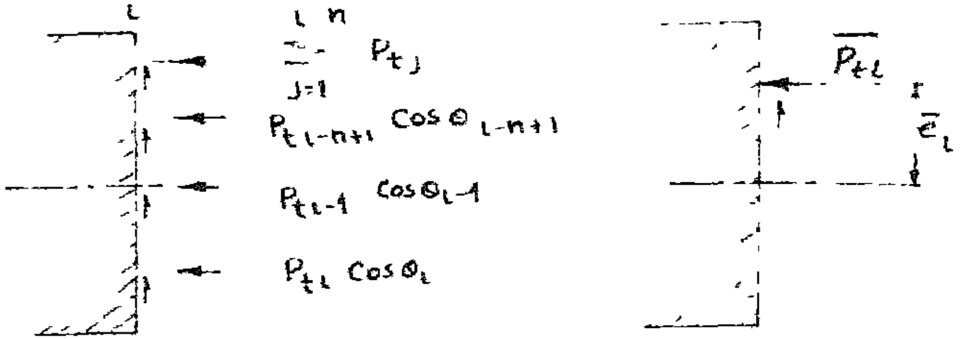

i
$\sum_{j=1}^{i\ n} P_{tj}$
$P_{t\,i-n+1}\cos\theta_{i-n+1}$
$P_{t\,i-1}\cos\theta_{i-1}$
$P_{ti}\cos\theta_i$
$\overline{P_{ti}}$
$\bar{e}_i$

FIG 2.6 - EQUIVALENT SECTION

and $$\bar{m}_i = \frac{\bar{P}_{ti}}{A_i \, \sigma_{cu}} \tag{2.38}$$

Eq.(2.36) can be rewritten as,

$$\bar{m}_i = \sum_{j=1}^{i-n} \frac{m_j \, A_j}{A_i} + \sum_{j=i-n+1}^{i} \frac{m_j \, A_j}{A_i} \cos \theta_j \tag{2.39}$$

$$\bar{m}_i = \sum_{j=1}^{i-n} m_{ij} + \sum_{j=i-n+1}^{i} m_{ij} \cos \theta_j \tag{2.40}$$

where

$$m_{ij} = \frac{m_j \, A_j}{A_i} = \frac{P_{tj}}{A_i \, \sigma_{cu}} \tag{2.41}$$

Taking moment of prestressing force about centroidal axis gives,

$$\bar{e}_i \, \bar{P}_{ti} = \sum_{j=1}^{i-n} P_{tj} \, (y_{ti}-d') + \sum_{j=i-n+1}^{i} P_{tj} \cos \theta_j \times \left\{ y_{ti} - \left[e_j - (i-j) \, a \tan \theta_j \right] \right\} \tag{2.42}$$

where d' = cover of the prestressing steel at top

Let $$\epsilon_i = e_i/h_i \tag{2.43}$$

Eq. (2.42) can be non-dimensionalized as,

$$\epsilon_i = \sum_{j=1}^{i-n} m_{ij} \frac{(y_{tt}-d')}{h_i} + \sum_{j=i-n+1}^{i} m_{ij} \cos \theta_j \times \left\{ \frac{y_{ti} - \left[e_j - (i-j) \, a \tan \theta_j \right]}{h_i} \right\} \tag{2.44}$$

where $\tan \theta_j = \frac{c_j - d'}{b}$ (2.45)

(b) Determination of α:

In cantilever bridges the area of cross section is varying longitudinally. The value of self weight bending moment coefficient (α) at any section is dependent on the cross sectional area variation. Two cases of cross sectional variation in longitudinal direction is considered.

(i) area of cross section varying linearly

(ii) area of cross section varying quadratically

(i) Area of cross section varying linearly:

Let q_0 and q_b be the weight per meter run of the sections corresponding to A_0 and A_b cross sectional areas at free and support ends respectively.

Considering i^{th} segment at a distance x from the free end (Fig. 2.7), let

A_i = area of cross section

M_{gi} = bending moment due to self weight at i^{th} segment

$$A_i = A_0 + (A_b - A_0)\, x/L \qquad (2.46)$$

From Fig. 2.7,

$$q = q_0 + (q_b - q_0)\, \xi/L$$

where $q_\xi = \gamma A_\xi$

Hence,

$$M_{g1} = \int_0^x q_\xi (x - \xi)\, d\xi$$

$$M_{g1} = \int_0^x \left[q_0 + (q_b - q_0)(\xi/L) \right] (x - \xi)\, d\xi$$

$$M_{g1} = q_0 \left[\frac{x^2}{2} - \frac{x^3}{6L} \right] + q_n \left[\frac{x^3}{6L} \right] \tag{2.47}$$

Non-dimensionalizing,

$$\frac{M_{g1}}{L^2 \gamma} = \frac{A_0}{2} \left[1 - 1/3 \left(\frac{x}{L}\right) \right] + \frac{A_n}{6} \left(\frac{x}{L}\right) \tag{2.48}$$

Let $\alpha_1 = \dfrac{A_1 L^2 \gamma}{M_{g1}}$

$$= \frac{A_1}{\frac{A_0}{2} \left[1 - \frac{1}{3} \left(\frac{x}{L}\right) \right] + \frac{A_n}{6} \left(\frac{x}{L}\right)} \tag{2.49}$$

$$\alpha \text{ at support (at } x=L) = \frac{6}{2\beta + 1} \tag{2.50}$$

$$\text{where } \beta = A_0/A_n \tag{2.51}$$

(ii) Cross sectional area varying quadratically:

Area of cross section of an i th segment may be expressed as,

$$A_i = A_0 + (A_b - A_0)\,(x/L)^2 \tag{2.52}$$

Eqs. (2.49) and (2.50) reduce to

$$\alpha_i = \frac{A_i}{\frac{A_0}{2}\left[1 - 1/6\left(\frac{x}{L}\right)^2\right] + \frac{A_n}{12}\left(\frac{x}{L}\right)^2} \qquad (2.53)$$

$$\sigma \text{ at } x = L = \frac{12}{5\beta + 1} \qquad (2.54)$$

Rearrangement of the above equations for an i^{th} segment are:

$$\bar{m}_i = \sum_{j=1}^{i-n} m_{ij} + \sum_{j=i-n+1}^{i} m_{ij} \cos\theta_j \qquad (2.55)$$

$$\theta_i = \sum_{j=1}^{i-n} m_{ij}\left(\frac{y_{ti} - d'}{h_j}\right) + \sum_{j=i-n+1}^{i} m_{ij} \cos\theta_j$$

$$\left\{\frac{y_{ti} - \left[c_j - (i-j)\, a \tan\theta_j\right]}{h_i}\right\} \qquad (2.56)$$

$$\underset{\text{(for linear variation)}}{\alpha_i} = \frac{A_i}{\frac{A_0}{2}\left[1 - 1/3\left(\frac{x}{L}\right)\right] + \frac{A_n}{6}\left(\frac{x}{L}\right)} \qquad (2.57)$$

$$\underset{\text{(for quadratic variation)}}{\alpha_i} = \frac{A_i}{\frac{A_0}{2}\left[1 - \frac{1}{6}\left(\frac{x}{L}\right)^2\right] + \frac{A_n}{12}\left(\frac{x}{L}\right)^2} \qquad (2.58)$$

The Eqs. (2.55) to (2.58) are used in the four

Eqs. (2.5) to (2.8) of permissible stresses. The values for ω, Δ_1, ρ_1 are to be suitably found out and substituted in Eqs. (2.5) to (2.8). The Eqs. (2.5) to (2.8) are to be modified when i is equal to one. Eqs. (2.55) and (2.56) are to be modified when i is less than n (i-n becomes negative) and i is greater than $(N_s - n + 1)$ (near the supports cable is not made inclined through n segments).

2.5 Minimum Weight Design:

Minimum weight gives minimum self weight moment and consequently maximum value of moment ratio $(R = M_a/M_g)$. Observing Eq. 2.13, R is maximum for maximum values of

(a) stress coefficients (b) ρ (c) ω (d) α

(a) Stress coefficients:

For any given value of h, ρ, and η the exact solution of the four Eqs. (2.5) to (2.8) with stress coefficients forces some restraint on the total solution. The value of which is defined as $\Delta = y_b/y_t$ depends upon the cross section finally selected. But based on permissible stresses is given by,

$$\Delta = \frac{\eta c_{ct} + c_{te}}{\eta c_{tt} + c_{ce}}$$

For full stress design the value of Δ is given by,

$$\Delta' = \frac{\eta c'_{ct} + c'_{te}}{\eta c'_{tt} + c'_{ce}} \qquad (2.59)$$

In case of long spans, the exact solution of the governing equations with maximum allowable coefficients might give an eccentricity exceeding the maximum eccentricity possible for that particular depth of the beam. In such case, the solution becomes absolutely ideal, giving a minimum weight design but violating the basic need of accommodating the steel within the section. It is seen from Eq. 2.17, for a particular m value eccentricity can be decreased by adjustment of c_{tt}, c_{ct}, α, ρ and ω. Depth factor is fixed in long span bridges. Efficiency ratio should not be adjusted since its effect on cross sectional area is severe. The value of α is dependent on R. Reducing c_{ct}, decreases the eccentricity. But it decreases the value of eccentricity limit as seen by Eq. (2.9). Only by reducing c_{tt}, the eccentricity required decreases and eccentricity limit is increased.

The shape factor (Δ') may or may not be equal to the Δ provided. On seeing Eq. (2.12) Δ can be decreased either reducing c_{ct} or c_{te}. If calculated shape factor (Δ) is less than the shape factor (Δ') needed, they can be matched by reducing either c_{tt} or c_{ce}. When the condition of providing steel inside the section is satisfied, reduction of c_{tt} results in lowering the prestressing steel below the required cover. The only alternative is to adjust c_{ce}.

The study of the equation leads to the conclusions given below and this can also be confirmed from Table 2.1. Table 2.1 gives the values of Δ, ϵ_{limit}, ϵ_{req} calculated from Eqs. (2.12), (2.9) and (2.18) respectively. The values in these equations are $\alpha = 4.0$, $L = 80.0$ metres , $\eta = 0.82$, $\sigma_{cu} = 530$ kg./cm^2 , $\rho = 2400$ kg./m^3 and $h = L/10$.

(i) If $\epsilon_{limit} < \epsilon_{req}$

Adjust c_{tt} ($c_{tt} < c'_{tt}$)

The rest c's max.

(ii) If $\Delta > \Delta'$

Adjust c_{ct} and c_{te}

Case a.

$c_{ct} < c'_{ct}$

The rest c's max. → eccentricity required increases.

Steel min.

Case b.

$c_{te} < c'_{te}$

The rest c's max. → eccentricity required decreases.

Steel max.

(iii) If $\Delta < \Delta'$

$c_{ce} < c'_{ce}$

The rest c's max.

(b) Section properties; Idealised sections

Most shapes of sections in actual construction are of unsymmetrical I, T, and box sections. For long spans of cantilever bridges, beams have normally heavier bottom flange. Present section discusses sectional properties of idealised sections for the sake of simplicity (11).

Properties of cross section of an idealised unsymmetrical I-section or box section (Fig. 2.8):

$$A = (b_t\ t_t + b_b\ t_b + h_w\ b_w) \tag{2.60}$$

$$= (b_t\ h)\left[c_t + c_f\ c_b + (1 - c_t - c_b)\ c_w\right]$$

$$= c_a\ (b_t\ h) \tag{2.61}$$

where

$$c_t = t_t/h, \qquad c_b = t_b/h$$

$$(2.62)$$

$$c_w = b_w/b_t, \qquad c_f = b_b/b_t$$

$$y_b = (1/A)\left[b_t\ t_t\ (h - \frac{t_t}{2}) + (h - t_t - t_b) \times b_w(\frac{h - t_t + t_b}{2}) + \frac{b_b\ t_b^{\ 2}}{2}\right]$$

$$= \left[\frac{c_t(1-\frac{c_t}{2}) + c_w\ (1-c_t-c_b)(\frac{1-c_t+c_b}{2}) + c_f\ \frac{c_b^2}{2}}{c_t+c_f\ c_b + (1-c_t-c_b)\ c_w} \right] h$$

$$= c_{yb}\ h \qquad (2.62)$$

$$y_t = h - y_b$$

$$= \left[\frac{c_b\ (1-\frac{c_b}{2})\ c_f + c_w\ (1-c_t-c_b)\ (\frac{1-c_t+c_b}{2}) + \frac{c_t^2}{2}}{c_t + c_f\ c_b + (1 - c_t - c_b)\ c_w} \right] h$$

$$= c_{yt}\ h \qquad (2.63)$$

$$I = b_t\ h^3 \left[\frac{1}{12} \left\{ c_t^3 + c_f\ c_b^3 + c_w\ (1-c_t-c_b)^3 \right\} + c_t\ (\frac{y_t}{h} - \frac{c_t}{2})^2 \right.$$

$$\left. + c_w\ (1-c_t-c_b)(\frac{1+c_t-c_b}{2} - \frac{y_t}{h})^2 + c_f\ c_b\ (\frac{y_b}{h} - \frac{c_b}{2})^2 \right]$$

$$= c_i\ b_t\ h^3 \qquad (2.64)$$

where,

c_a, c_i, c_{yb}, c_{yt} are non-dimensional coefficients as given in the above expressions.

$$\Delta = \frac{y_b}{y_t} = \frac{c_t(1-\frac{c_t}{2}) + c_w\ (1-c_t-c_b)(\frac{1-c_t+c_b}{2}) + c_f\ \frac{c_b^2}{2}}{c_f\ c_b\ (1-\frac{c_b}{2}) + c_w\ (1-c_t-c_b)(\frac{1-c_b-c_t}{2}) + \frac{c_t^2}{2}} \qquad (2.65)$$

$$\rho = c_i/c_a \qquad (2.66)$$

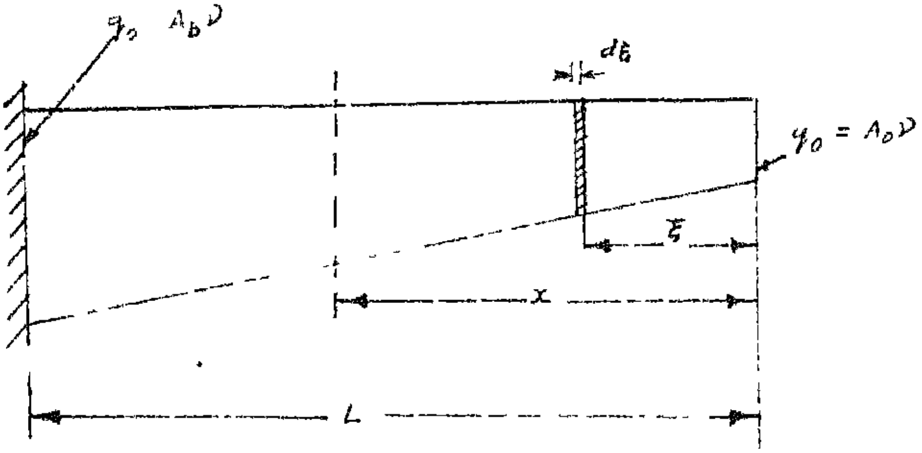


FIG 2.7 - CANTILEVER BEAM

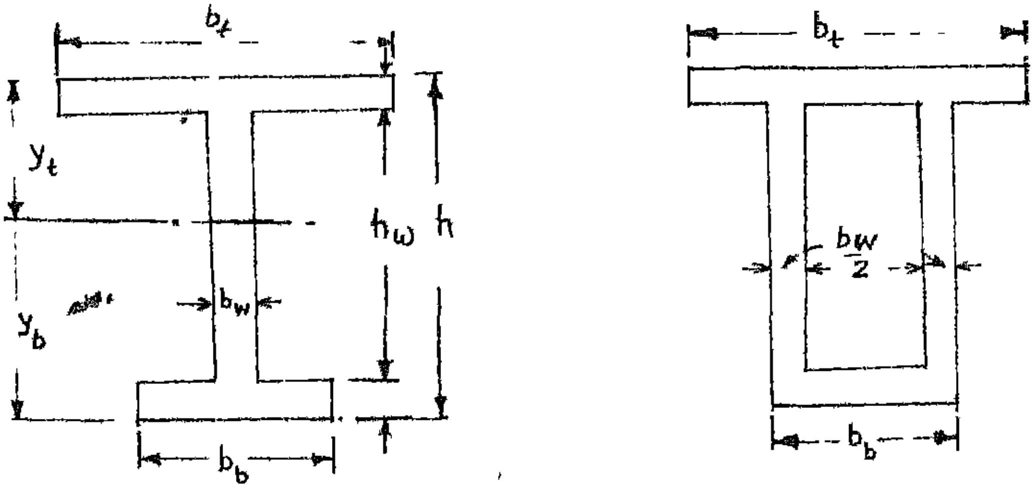


FIG 2.8 - IDEALISED SECTIONS

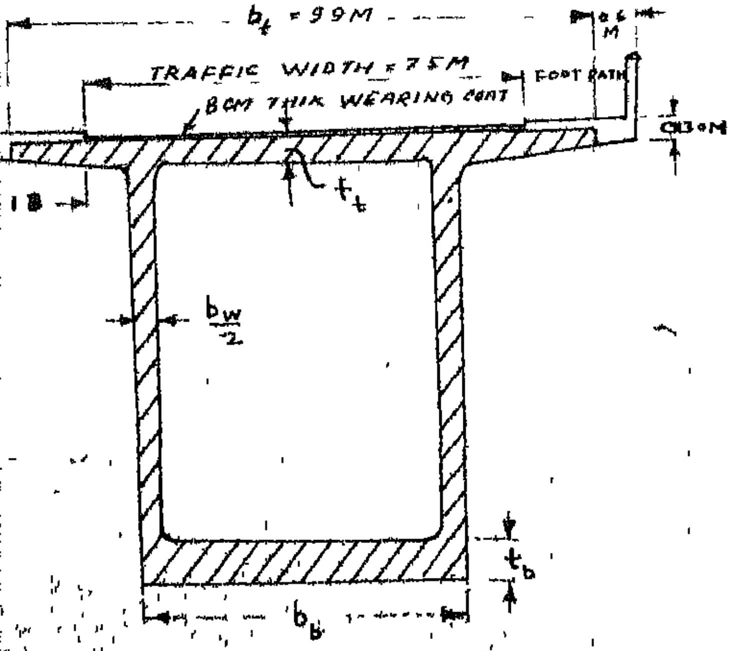


FIG 2.9 - TWO LANE BRIDGE CROSS SECTION WITH FOOT PATH

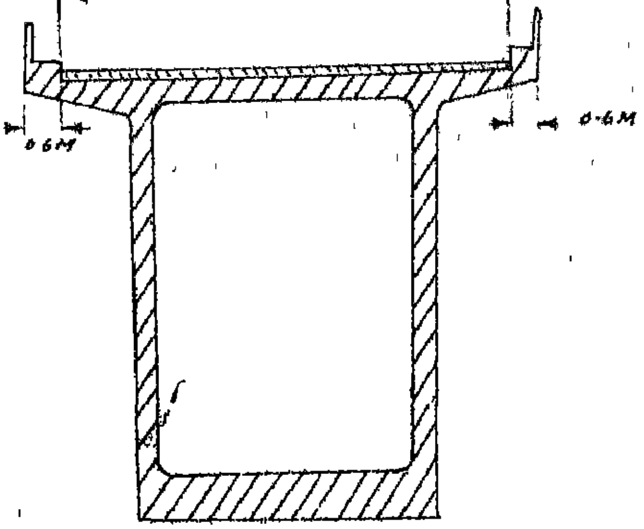


FIG 2.10 - TWO LANE BRIDGE SECTION WITHOUT FOOTPATH

TABLE 2.1: STRESS COEFFICIENTS:

c_{te}	c_{cT}	c_{ce}	c_{tt}	Δ	ϵ_L	ϵ_{req}
0.018	0.420	0.274	0.018	0.794	0.507	0.830
0.018	0.420	0.274	-0.021	0.704	0.537	0.755
0.018	0.420	0.274	-0.041	0.659	0.552	0.719
0.018	0.420	0.274	-0.041	0.659	0.552	0.719
0.018	0.420	0.274	-0.041	0.659	0.552	0.719
0.018	0.420	0.264	-0.041	0.632	0.563	0.733
0.018	0.420	0.254	-0.041	0.604	0.573	0.749
0.018	0.420	0.254	-0.041	0.632	0.563	0.749
0.018	0.380	0.254	-0.041	0.663	0.551	0.750
0.018	0.380	0.254	-0.041	0.663	0.551	0.750
0.0081	0.380	0.254	-0.041	0.685	0.544	0.740
-0.0018	0.380	0.254	-0.041	0.707	0.534	0.729

TABLE 2.2: SECTIONAL PROPERTIES:

c_f	c_w	c_t	c_b	Δ	ρ
0.50	0.060	0.04	0.05	1.291	0.1518
0.50	0.080	0.04	0.05	1.251	0.1413
0.50	0.100	0.04	0.05	1.221	0.1331
0.50	0.100	0.04	0.05	1.221	0.1331
0.60	0.100	0.04	0.05	1.144	0.1373
0.70	0.100	0.04	0.05	1.076	0.1406
0.80	0.100	0.04	0.05	1.015	0.1434

A simple illustration of the behaviour of ρ is shown in Table 2.2 with assumed values of c_b, c_t, c_f and c_w. The value of ρ should be increased to increase the value of R. It can be seen from the table, ρ is increased by

(i) decreasing the web ratio ($c_w = b_w/b_t$)

(ii) increasing the flange ratio ($c_f = b_b/b_t$)

2.6. Design Procedure:

A detailed computer program has been developed based on the equations derived for minimum weight design of cantilever bridges of two lane traffic width. A detailed flow diagram of the program is given in Appendix I. The following assumptions are made in the design problem.

1. Box section (Figs. 2.9 and 2.10) is chosen on account of its high torsional rigidity.

In the case of Bassein Creek bridge this assumption was verified analytically (12) at mid span, by comparing the vertical displacement of the box which was subjected to eccentric live loads, with vertical deflection of the box due to longitudinal bonding. It was found that the vertical displacement of the web due to rotation of the box was less than 1 percent of the vertical deflection due to longitudinal bending.

b. Hinge is provided at the junction between the cantilevers.

c. The effective prestressing force is η times that of transfer. The value for η assumed is 0.82.

d. The depth of beam at support is pier spacing/20 (2L/20) and at the centre 2L/60. The cross section area at the cantilever free end is calculated on minimum requirements. It is usual practice to provide 15 cm minimum thickness of top flange and thickness is increased in accordance with the traffic width. Minimum bottom thickness and the web thickness normally adopted are given below.

$$t_t = b_t/100.0 + 0.15 \quad \text{(dimension in metres)}$$

$$t_b = 0.15$$

$$t_{w1} = 2.0\ (0.10 + h_b/40.0)$$

$$t_{w2} = 0.15 + b_t/20.0$$

$$t_w = \text{maximum of } t_{w1} \text{ and } t_{w2}.$$

e. Loadings:

1. Live Load:

(i) IRC class AA loading or two lane class A loading whichever produces more severe condition is considered.

(ii) Impact factor 0.08 is added to the loading.

(iii) Live load on footpath (Fig. 2.9) is 400 kg./m^2.

2. Dead Load:

(i) Wearing coat of 8 cm thick is adopted.

(ii) Weight of railing on each side is 100 kg./m.run.

(iii) Weight of cantilever foot-path (Fig. 2.9) = 2 x 1000 kg/m.run.

Weight of cantilever kerb (Fig. 2.10) = 2 x 432 kg/m. run. The above loads act on the structure at working load condition.

f. Permissible stress coefficients are according to IS-1343-1960.

g. Excessive prestressing steel causes primary failure at ultimate due to crushing of concrete. Very low percentage of steel causes fracture of steel. Lower and upper limits of steel to avoid these failures are calculated as given in IS-1343 clause 6.8.2.2. The ultimate moment capacity is calculated according to clause 6.8.3 of I.S. The ultimate moment capacity of the beam should not be less than the ultimate moment required. The ultimate moment required is calculated by assuming dead and live load factors 1.5 and 3.0 respectively.

CHAPTER III

RESULTS

3.1 Introduction:

The effect of several parameters on the weight of the cantilever bridge is studied from the results of the computer program. The value of η (P_e/P_t), σ_s' (ultimate tensile strength of prestressing steel) and the ratio of cover distance to the centre of steel (d'/h) used in the program are 0.82, 1400 kg/cm^2 and 0.06 respectively. Unless stated otherwise, all the results given are for a typical bridge section shown in Fig. 2.9 (7.5 m. traffic width with 1.8 m wide footpath), h/L ratio at the support is equal to 1/10 and the permissible stresses used are those of loads likely to increase of IS-1343, 1960. An optimization program with the necessary constraints will give a minimum weight design. The main object of the present investigation is to help the designer to get an idea of the design efficiency based on various parameters. The selection of various parameters based on practical limitations plays an important part. Therefore, if a designer has an idea of the effect of various parameters not merely of qualitative nature but also of quantitative nature, then the number of trials to get an economical design will be the least.

3.2. Variation of Area of Cross Section at Support With Respect to Span:

In cantilever bridges, the bending moment curve due to self weight is of third or fourth degree. To start with, the area of cross section at the free end is assumed based on practical limits. The section at the support is the most critical one and is designed on the basis of the computer program outlined in Chapter II. After the sections at support and the free end are fixed, a suitable profile for the depth of the beam, area of prestressing steel and eccentricity are chosen so that all the sections are within the safety limit.

Figs. 3.1 and 3.2 indicate the behaviour of the area of cross section at support due to increase in span. Two cases of variation of the areas of cross section along the span are considered.

(i) linear variation of area of cross section

(ii) quadratic variation of area of cross section

The area tends to increase exponentially for long limiting span. The rate increases with the decrease in strength of concrete and when the variation of area of cross section is assumed to be linear. For example, for a span of 80 m using M 530 mix, the increase in area of cross section is about 8 percent in a linear variation of self weight as compared to the increase of 11.5 percent for 100 m span. When the

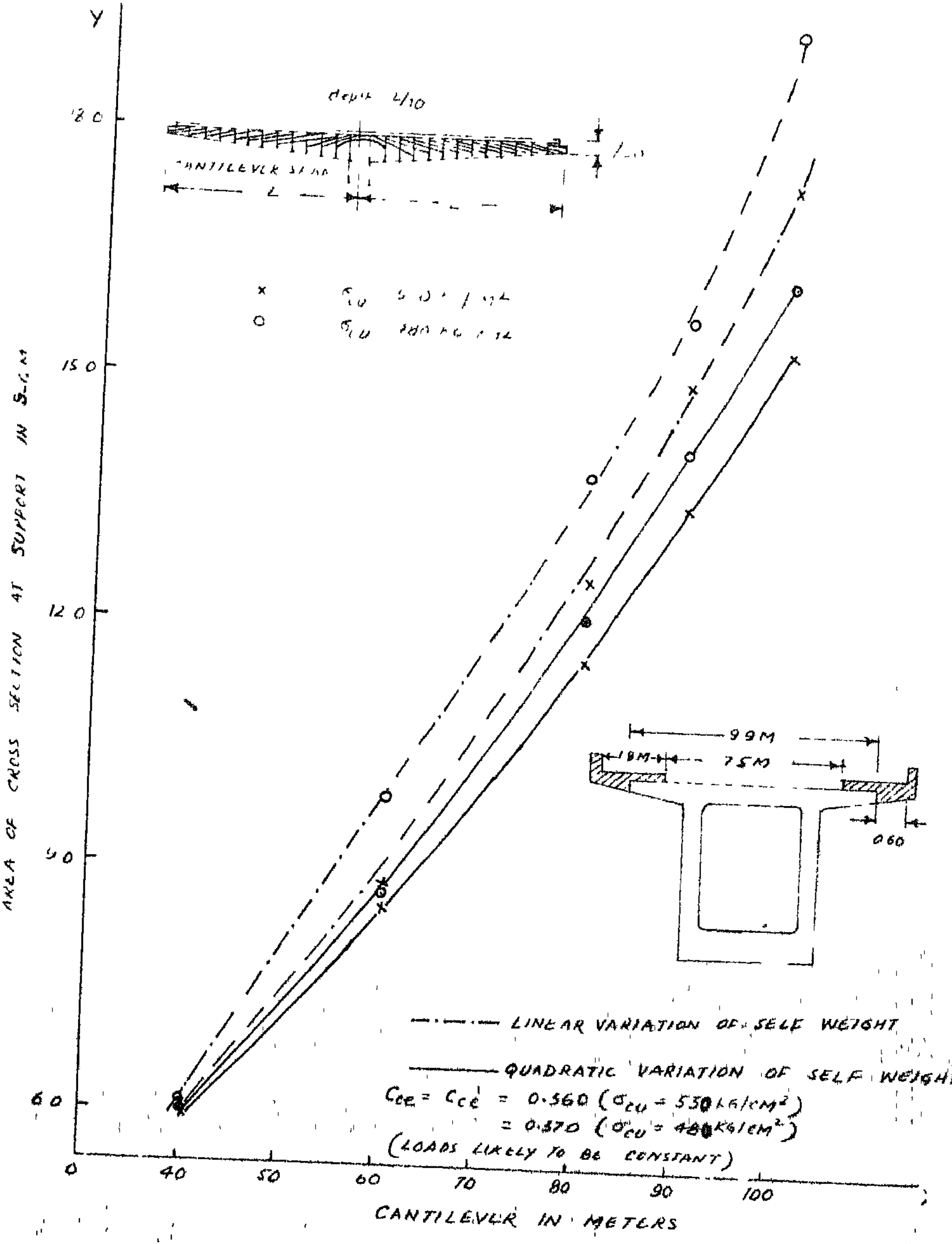


FIG.31. AREA OF CROSS SECTION AT SUPPORT WITH SPAN

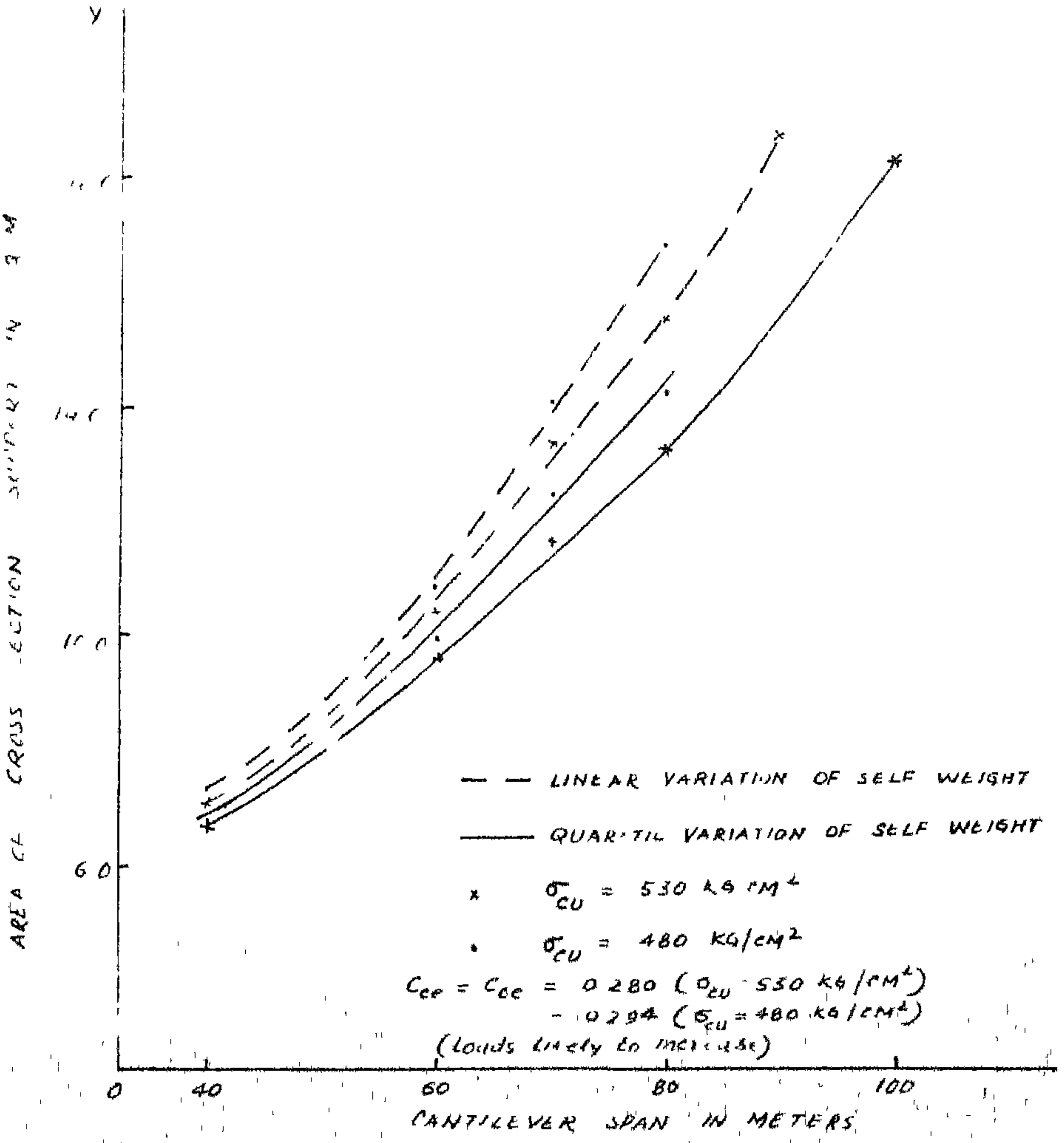


FIG 52 — AREA OF CROSS SECTION AT SUPPORT WITH SPA

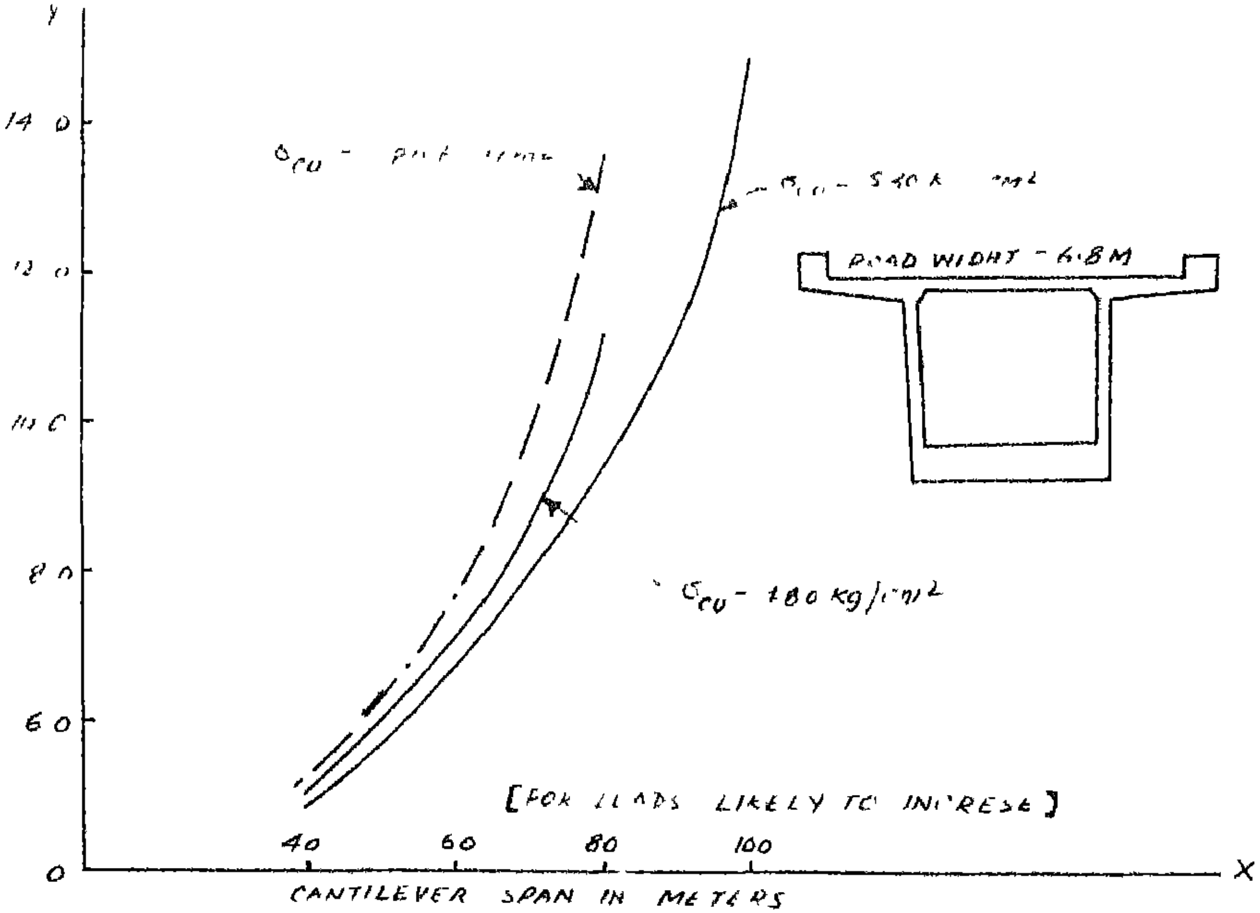


FIG 3.5 — AREA OF CROSS SECTION AT SUPPORT WITH SPAN

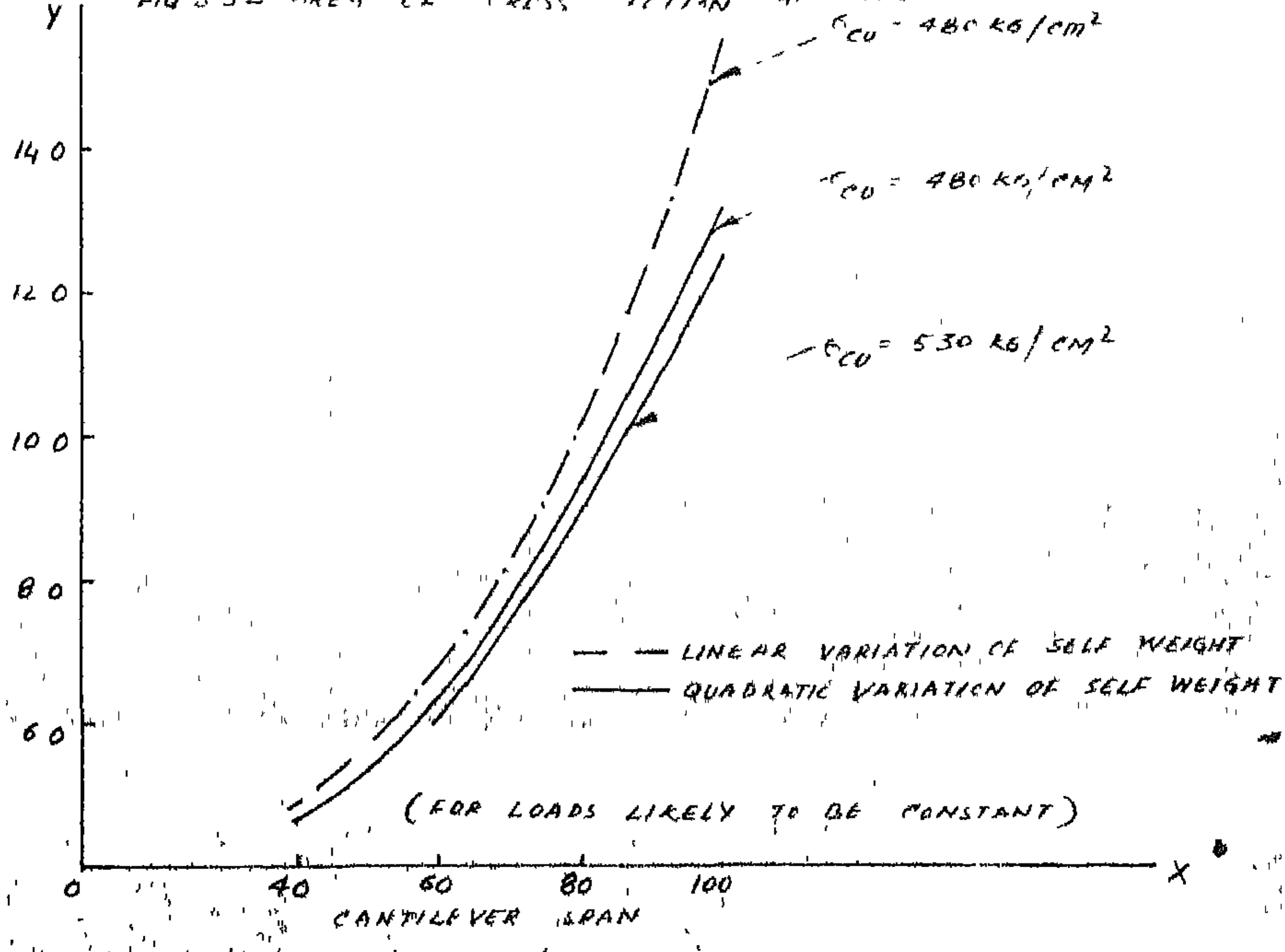


FIG 3.4 — AREA OF CROSS SECTION AT SUPPORT WITH SPAN

concrete strength is decreased to 480 kg/cm^2, the percentage increases are 12 and 18.5 respectively. Fig. 3.1 corresponds to the design for the loads likely to increase and Fig. 3.2 corresponds to the design for the loads not likely to increase. The effect of change in the value of c'_{ce} on the area of cross section is apparent.

Figs. 3.3 and 3.4 are drawn similar to Figs. 3.2 and 3.1 respectively for the bridge section shown in Fig. 2.10. The limiting spans shown by Figs. 3.2 and 3.3 are comparable to the analytical results derived in Chapter II.

3.3. Effect of the Area at the Free End:

The selection of the area at the free end has a significant effect on the area of cross section at the support when the area of cross section is varying either linearly or quadratically from the free end to the support (Fig.3.5). This affects the value of the bending moment coefficient (α), hence the area of cross section at the support. The percentage increase in the area of cross section at the support is less as compared to that at free end.

The section at the free end is designed subject to practical limitations. The deck slab is designed to the transverse bending moment. The bottom flange, web, and the height of the section are selected based on practical

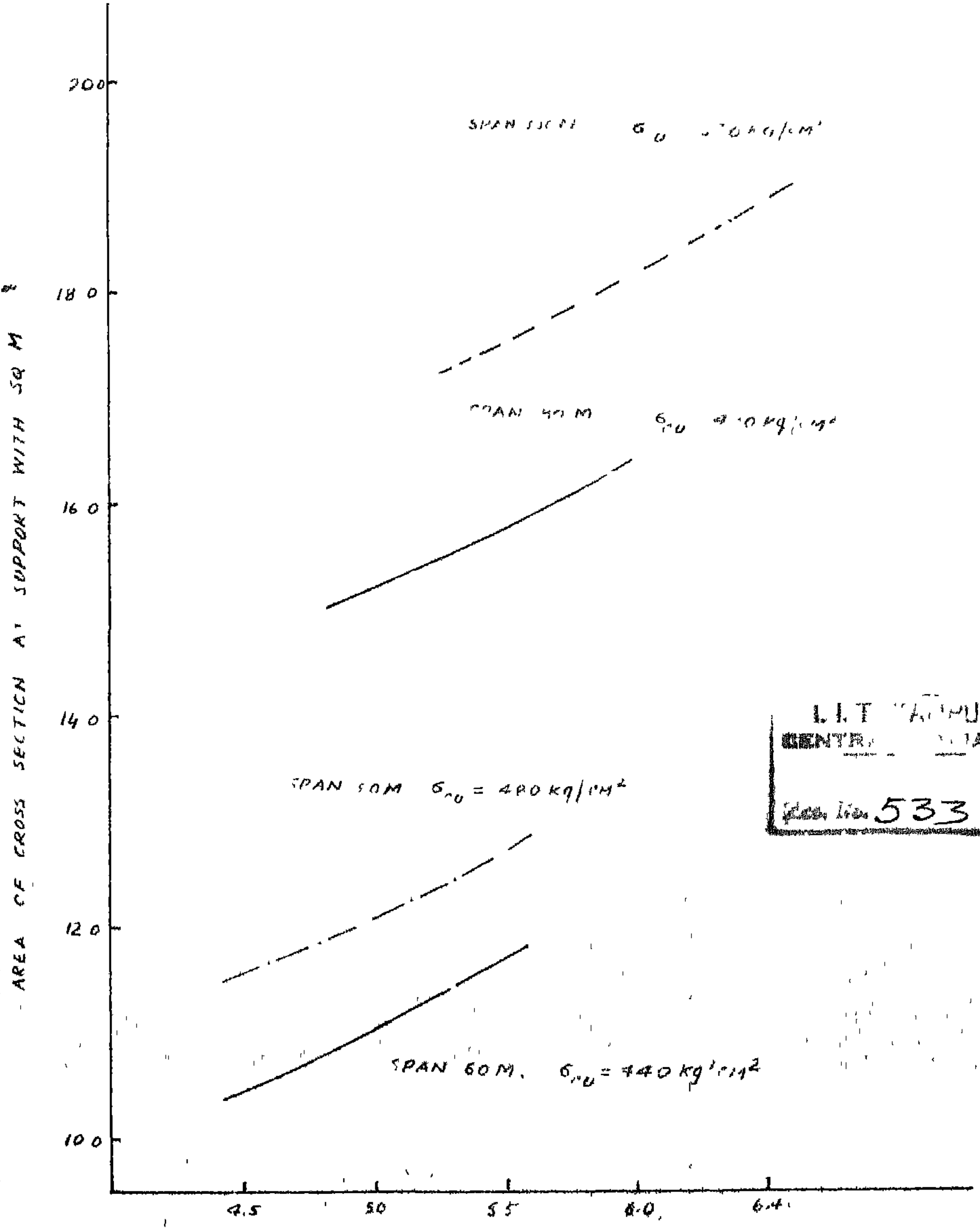


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FIG.5.5 — AREA OF CROSS SECTION AT SUPPORT WITH AREA OF CROSS SECTION AT FREE END.

considerations. The overall section is checked for the prestressing force at the end section.

3.4. Effect of Strength of Concrete:

Variation in the area of cross section with strength of concrete is shown in Fig. 3.6. As the strength of concrete increases, the area of cross section decreases. This straight forward conclusion could be made even without the graphs. The graphs not only illustrate the qualitative nature, but also the quantitative values which are helpful in selecting the concrete strength. It is desirable to use a high strength concrete. However, the cost of the high strength concrete is more, and hence a proper selection of strength of the concrete is to be made such that the cost of the structure is minimum. Other limitations in selecting a high strength concrete are the workability, availability of proper equipment and skilled manpower for good and uniform compaction. For an economic design, the longer the span the higher should be the strength of the concrete.

3.5. Effect of Depth of Beam at the Support:

Fig. 3.7 indicates the effect of depth on the area of cross section for a given cube strength. As the depth increases (from L/11 to L/9), the area of cross section

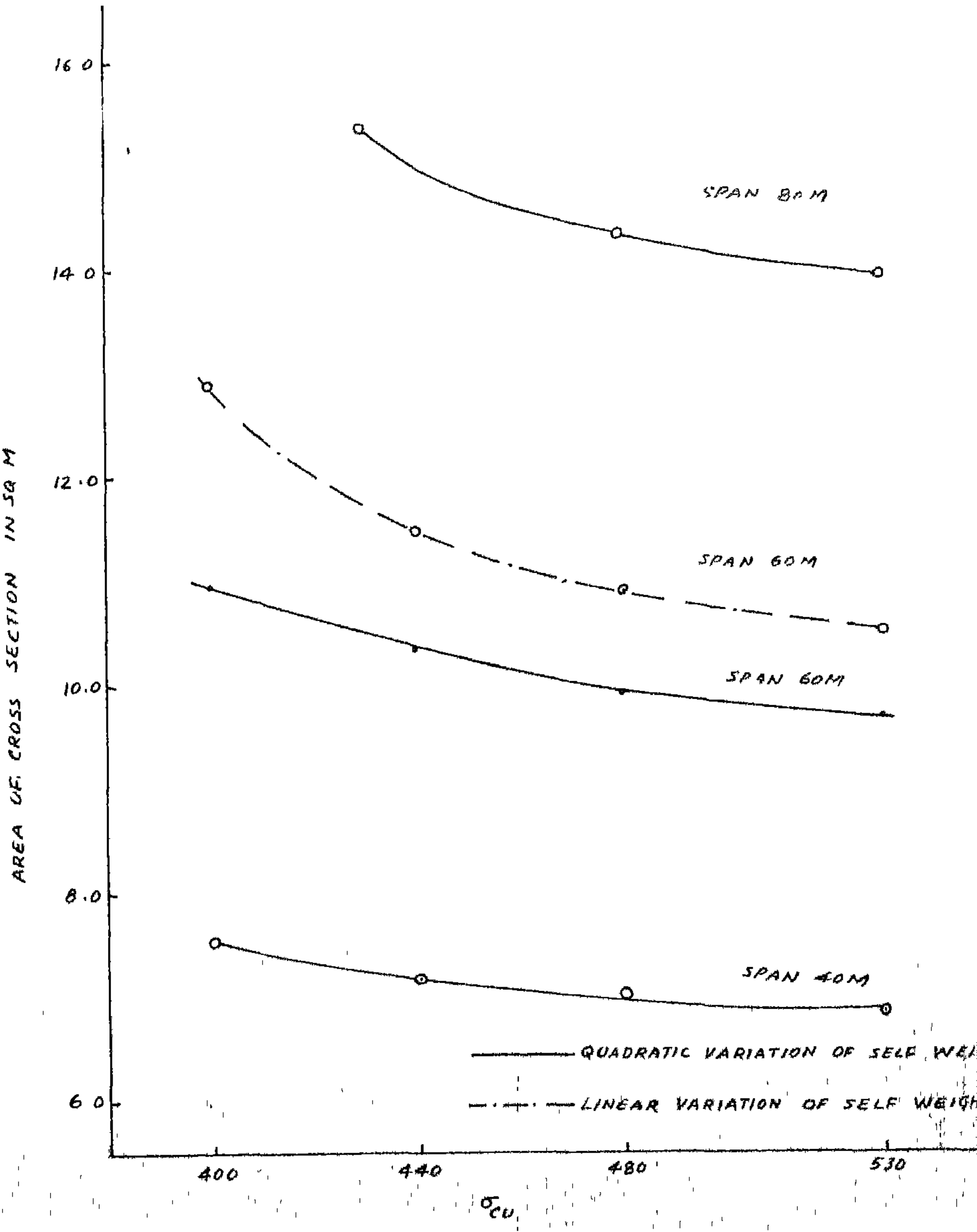


FIG. 3.6 — AREA OF CROSS SECTION AT SUPPORT WITH CUBE STRENGTH

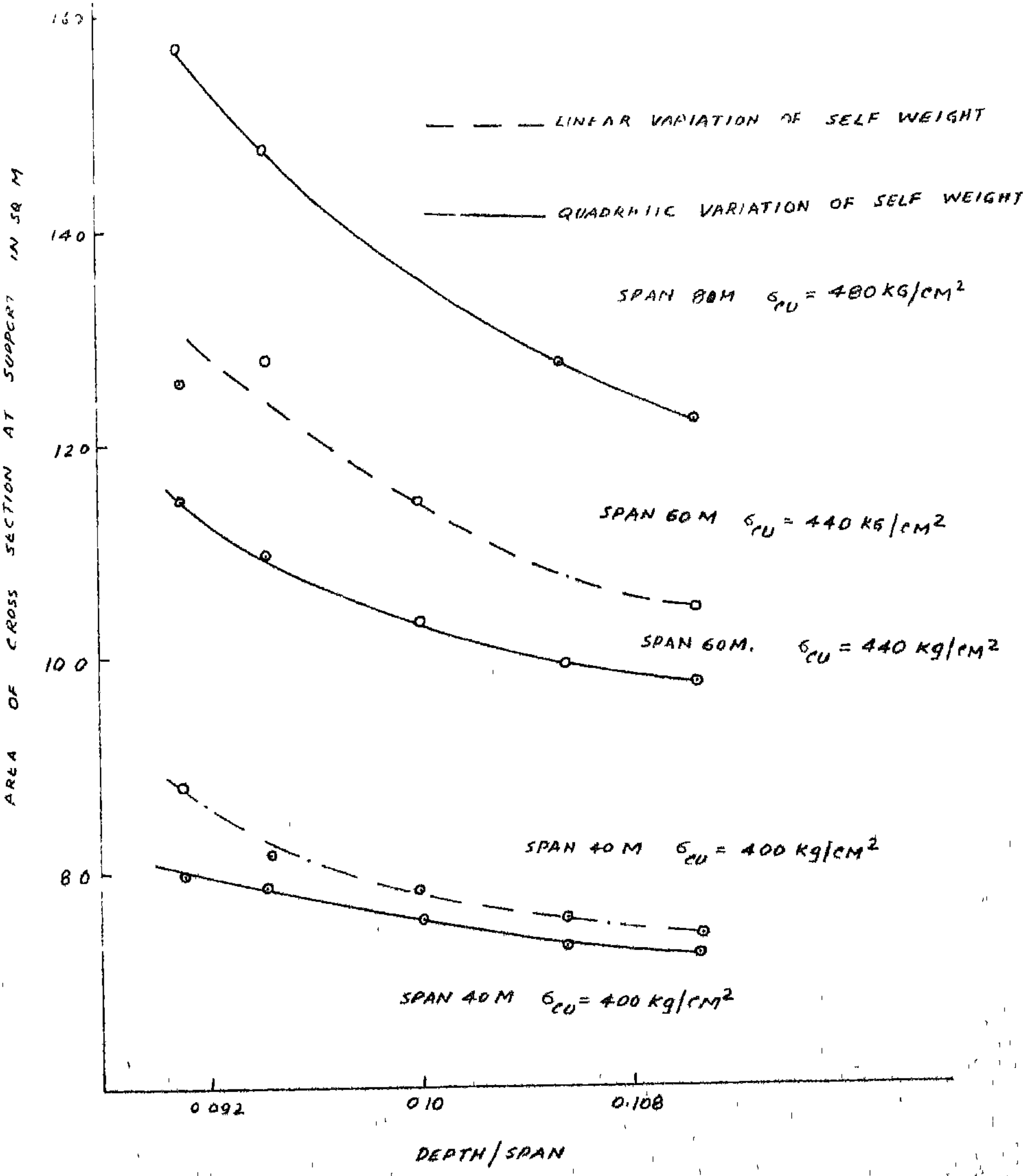


FIG 3 7 – AREA OF CROSS - SECTION WITH (h/L) decreases. The decrease in the area of cross section with increase in depth is faster as compared to the increase in concrete strength. The increase in the area of cross section with decrease in height is more pronounced in the case of linear variation of area of cross section than that of quadratic. The effect of depth varying from L/9 to L/11 on area of cross section is more in the case of long span bridges. The increase in the area is about 22 percent for a span of 80 m, whereas it is only 9 percent in the case of 40 m span when the depth is decreased from L/9 to L/11.

It is desirable to provide a higher depth but again the depth is subjected to architectural and practical limitations. In the case of long spans, the exact solution of the governing equations (Eqs. 2.5 to 2.8) with full stress condition gives an eccentricity exceeding the maximum eccentricity feasible if span to depth ratio is adopted as 10. However, the eccentricity as calculated by the governing equations is adjustable by lowering the permissible tensile stress at transfer (c_{tt}).

3.6. Effect of Efficiency Ratio (ρ):

Fig. 3.8(a) illustrates the effect of flange ratio ($c_f = b_b/b_t$) on the cross sectional area. The decrease in the flange ratio reduces the efficiency ratio and in turn the cross sectional area is increased (Fig. 3.8(b)).

A procedure was suggested (8) for the minimum weight design of simply supported beams with the assumption of maximum efficiency ratio. In that procedure the value of the shape factor and area of cross section were obtained. The coefficients of the section c_f, c_b c_w and c_t were chosen for that value of shape factor and efficiency ratio. But efficiency ratio cannot be assumed in the case of bridges with box section because of the constraints imposed on the top flange thickness, governing for the transverse bending moment and other practical limits. To make the bending moment minimum, the webs should be at 0.207 b_t from the ends. Even though a starting value of efficiency ratio is taken, it is successively corrected to suit the other constraints.

It is desirable to select a maximum efficiency ratio. Efficiency ratio can be increased by choosing larger symmetric flange areas and a slender web. As the span increases, the bottom flange becomes heavier at the support and the efficiency ratio is decreased. It appears to be reasonable to have the value of efficiency ratio in the range of 0.135 to 0.145 for spans from 100 m to 70 m.

3.7. Behaviour of Self Weight Bending Moment Coefficient ($\alpha = M_g/\nu A L^2$):

The variation of area of cross section along the span

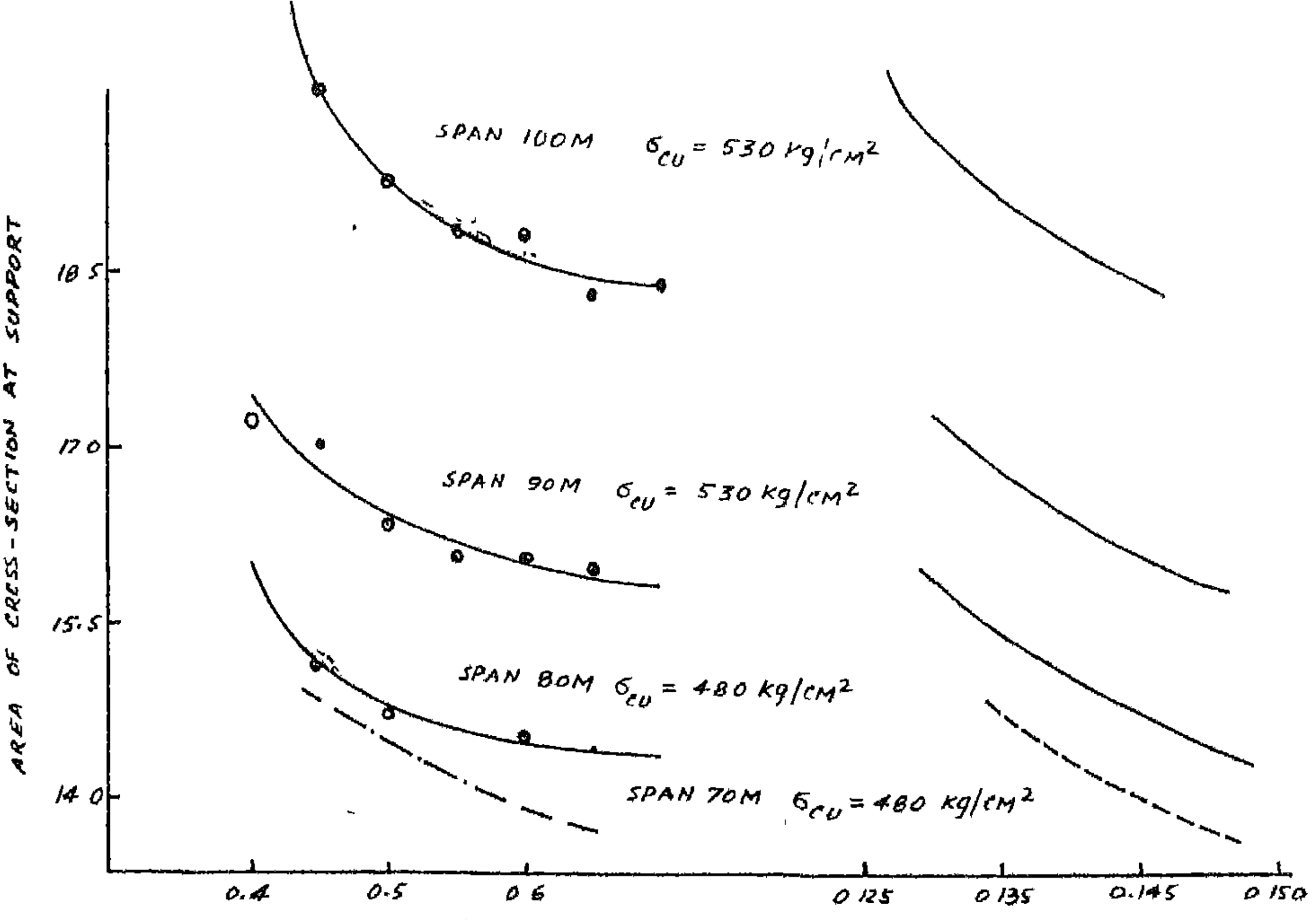


FIG 3.8.a – AREA OF CROSS SECTION AT SUPPORT WITH FLANGE RATIO

FIG 3.8 b – AREA OF CROSS-SECTION A
SUPPORT WITH ρ

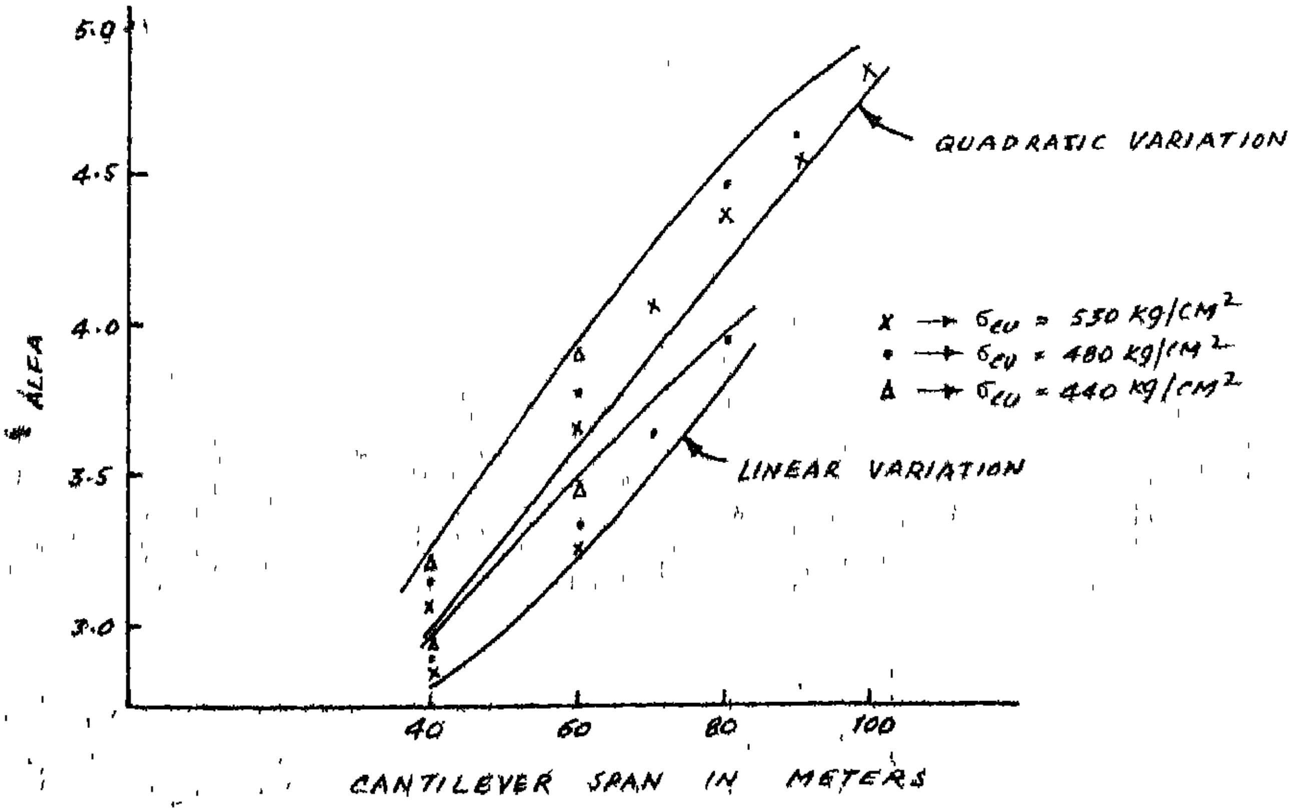


FIG. 3.9 – ALFA WITH SPAN

fixes the value of the self weight bending moment. For example, the values of α's of a simply supported beam and a cantilever of uniform section are 8 and 2 respectively. It is seen from Eq. 2.13 that the self weight bending moment is minimum for the maximum value of α. The value of α can be increased by assuming a suitable variation of area of cross section along the longitudinal axis. Fig. 3.9 gives the values of α for different strengths of concrete and for the cases of variation of area of cross section as linear and quadratic. Once the area at the free end is fixed the value of α decreases with the increase in cube strength.

The value of α is dependent on the moment ratio ($R = M_a/M_g$). The values of α are calculated from Eqs. 2.57 and 2.58 for linear and quadratic variations of the self weight respectively. The calculated value of α should agree with the starting value of α using which the moment ratio (R) is calculated. If there is difference between the starting and the calculated values of α, then the starting value is successively corrected till the difference between the two is within the permissible range.

Reasonable values of α, approximated from Fig.3.9 can be given in terms of span as following:

(i) for area of cross section varying linearly

(upto 80 m), $\alpha = 2.940 + 0.025\ (L - 40)$

(ii) for area of cross section varying quadratically (upto 100 m), $\alpha = 3.125 + 0.029\ (L - 40)$

3.8. Behaviour of Shape Factor ($\Delta = y_b/y_t$):

The variation of shape factor with respect to span and for two different strengths of concrete is illustrated in Fig. 3.10. As the span increases the bottom flange becomes much heavier. This causes the centroidal axis to shift down and hence the value of shape factor decreases. The shape of the section fixes the working-stresses. It is important to choose a proper shape factor. The value of Δ is between 1.15 and 0.65 for spans from 40 m to 100 m with a concrete strength of 530 kg./cm^2. The values decrease further with decrease in concrete strength.

3.9. Moment Ratio ($R = \dfrac{M_s + M_L}{M_g}$):

The self weight bending moment is not known to the designer to start with. Moment ratio is calculated from the solutions of the governing equations. The area of cross section at support is calculated from the value of R and α. An illustrative behaviour of the moment ratio is given in Fig. 3.11. with increase in span. The value of moment ratio increases from 0.45 to 0.80 as the span decreases from 100 to 40 m. When the self weight variation is linear the moment ratio

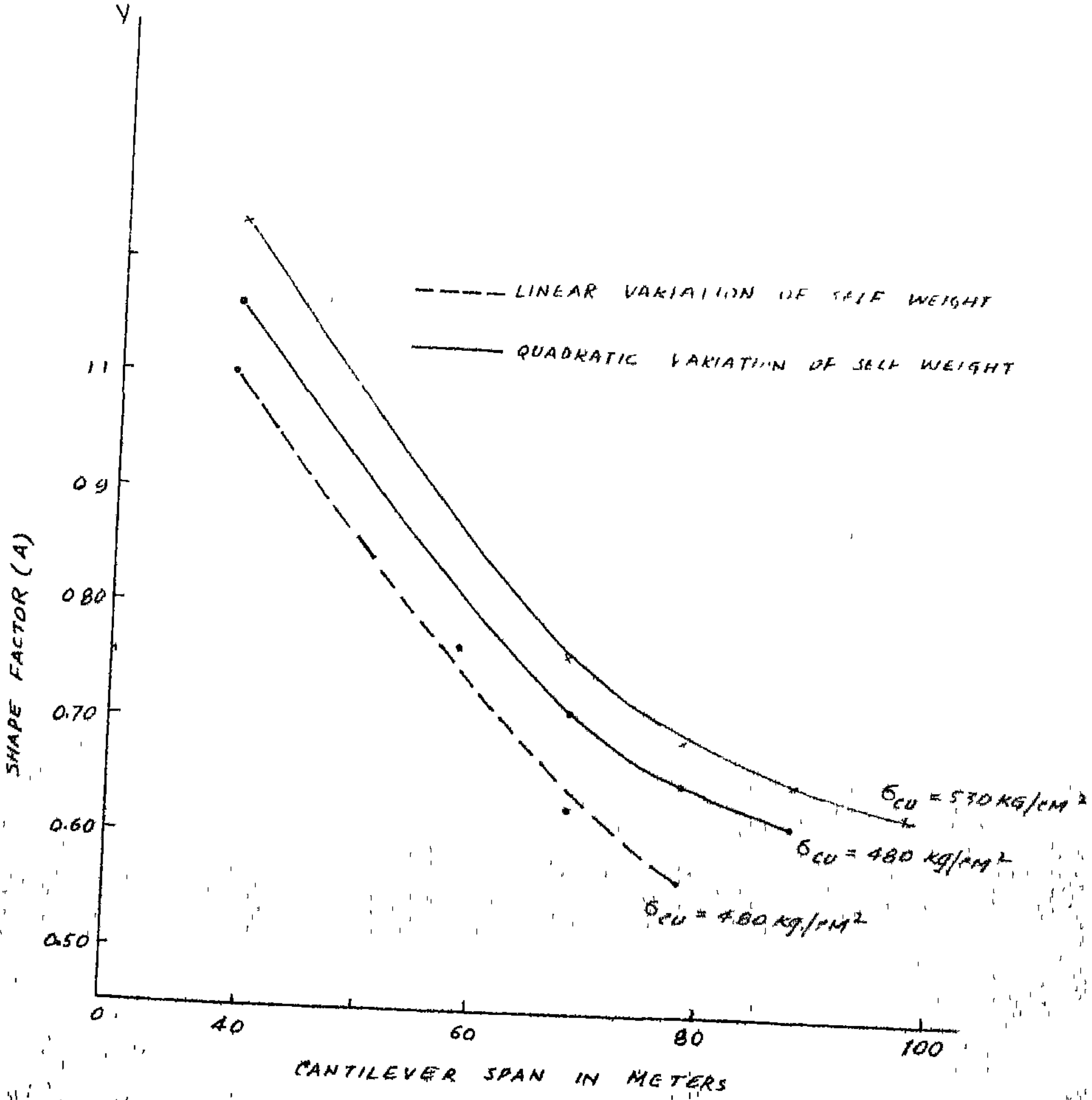


FIG. 3.10 – SPAN Vs Δ

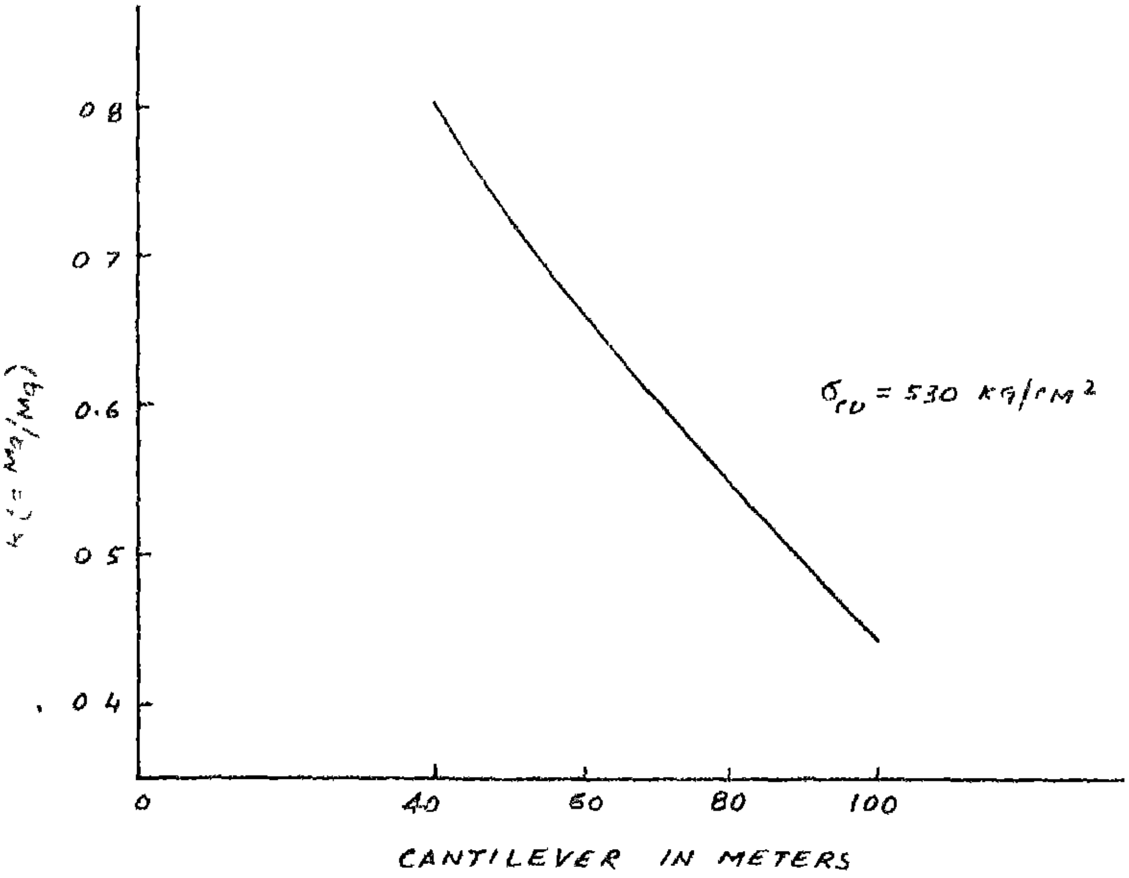


FIG 3.11 - MOMENT RATIO WITH SPAN

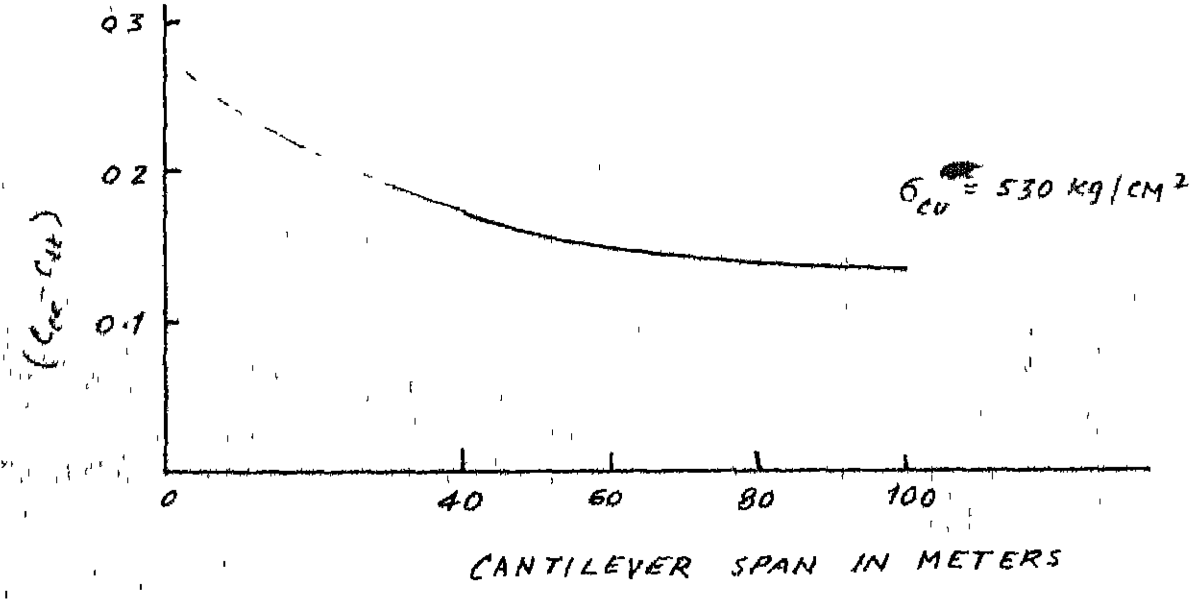


FIG. 3.12 - $(C_{ce} - C_{tt})$ WITH SPAN

decreases further because of the decrease in the value of σ.

3.10. Influence of Stress Coefficients on Minimum Weight Design:

Figs. 3.12 to 3.15 illustrate the effect of the working stresses on the area of cross section and the prestressing force. Fully stressed design is not feasible in long span bridges. The shape factor decreases as the span increases (Fig. 3.10) and this shifts the centroidal axis towards the bottom flange. The centroidal axis near the bottom flange reduces the difference of actual feasible stresses at bottom fibre (c_{ce} - c_{tt}). But it increases the difference of actual feasible stresses at top fibre (c_{ct} - c_{te}).

A typical behaviour of (c_{ce} - c_{tt}) with span is illustrated in Fig. 3.12. In minimum weight design, c_{ce} is kept equal to the maximum permissible value. The value of c_{tt} is adjusted to accommodate the steel within the section. The effects of other two coefficients c_{ct} and c_{te} are investigated. The typical effects of (c_{ct} - c_{te}) on the area of cross section and the steel ratio ($m = P_t/A\ \sigma_{cu}$)(or the steel percentage) required at support are illustrated in Figs. 3.12, 3.13 and 3.14.

When the Δ calculated by the working stress coefficients is less than the Δ obtained from the value of R, either one of the coefficients (c_{ct} and c_{te}) or combination of the these two has to be reduced. Accordingly for every span, there is a range of the difference of the values c_{ct} and c_{te}. The minimum value of (c_{ct} - c_{te}) can be achieved by adjusting c_{te} only and keeping c_{ct} equal to c'_{ct}. The maximum value can be obtained by adjusting c_{ct} and keeping c_{te} equal to c'_{te}. The intermediate values are given by the simultaneous reduction of both the coefficients.

A minimum weight solution is arrived at where the value of (c_{ct} - c_{te}) is maximum. The decrease in the area of cross section is achieved at the cost of increase in the percentage of steel. The quantity of steel can be obtained from the Fig. 3.15. In Fig. 3.13, lines A_1 A_1, A_2 A_2, A_3 A_3 and A_4 A_4 indicate a deccase in the areas of cross section when the value of (c_{ct} - c_{te}) increase, for a span of 40 m, 70 m, 80 m, and 90 m respectively. The corresponding steel percentages or steel ratio required are represented by lines m_1 m_1, m_2 m_2, m_3 m_3 and m_4 m_4.

The IS-1343 Code specifies an upper limit for the amount of steel to be provided. The upper limits of steel are calculated for the computed sections and are shown by points a, b, c and d in Fig. 3.13. The points a, b, c, and d lie approximately in the straight line l_1 l_1. The lines dropped from a, b, c, and d cut the curves A_1 A_1, A_2 A_2, A_3 A_3 and A_4 A_4 at a_1, b_1, c_1, and d_1 respectively. The portions of the curves A_1 A_1, A_2 A_2, A_3 A_3 and A_4 A_4 above the curve l_2 l_2

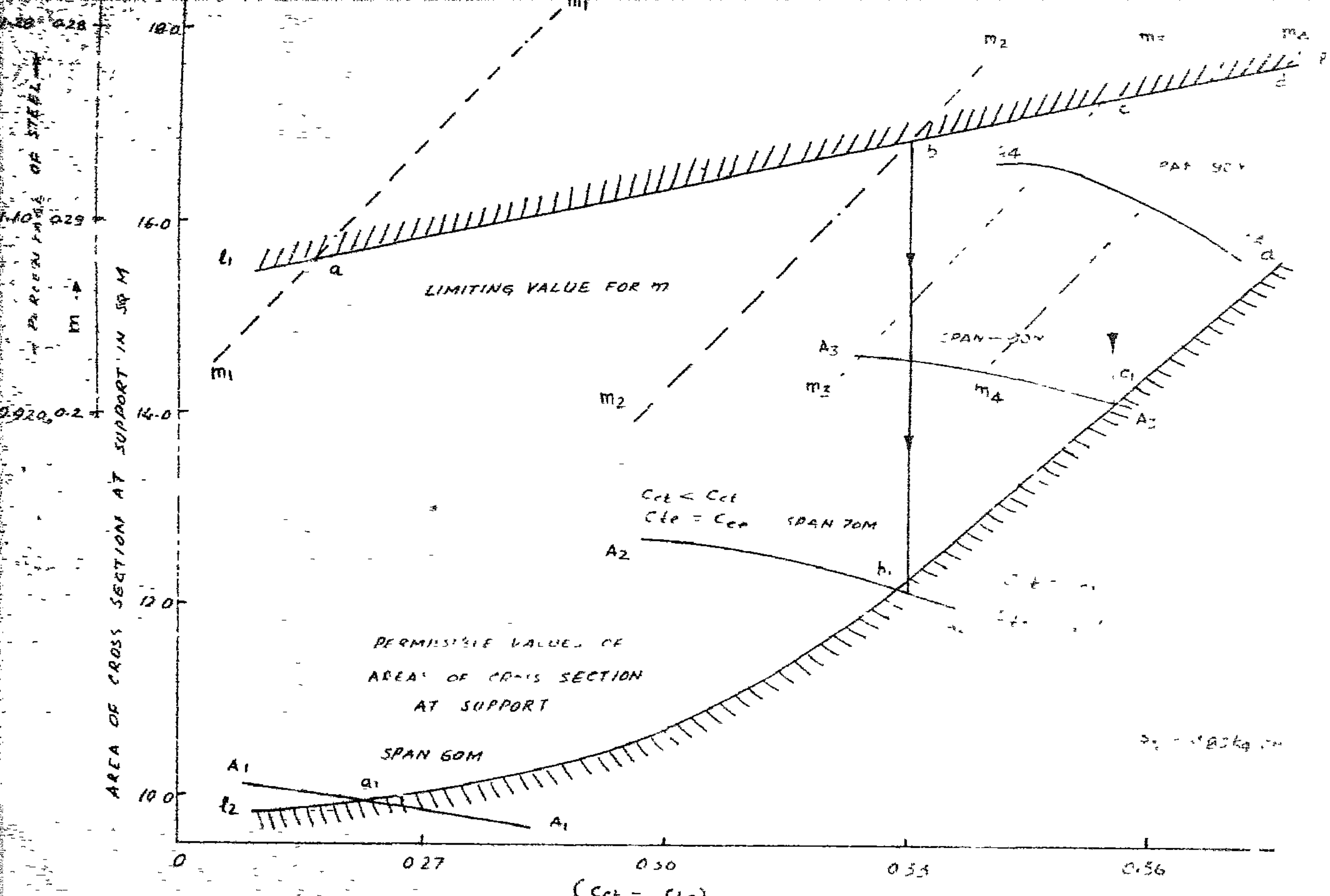
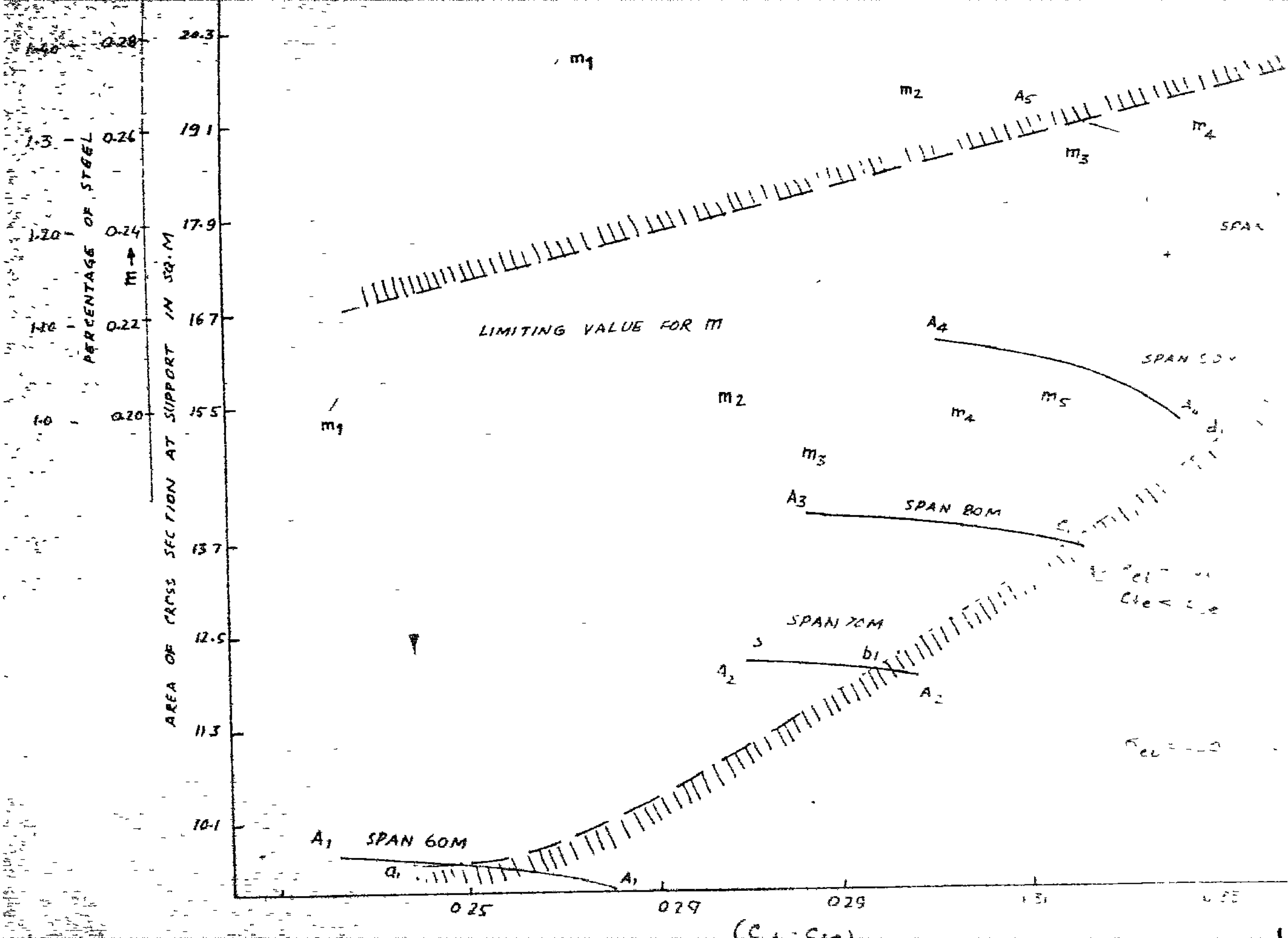


FIG-3.13 — AREA OF CROSS SECTION AT SUPPORT WITH (Cct - Cte)


PERCENTAGE OF STEEL
m →
AREA OF CROSS SECTION AT SUPPORT IN SQ.M
1.40
1.3
1.20
1.10
1.0
0.28
0.26
0.24
0.22
0.20
20.3
19.1
17.9
16.7
15.5
13.7
12.5
11.3
10.1
LIMITING VALUE FOR m
m1
m2
m3
m4
m5
A1
A2
A3
A4
A5
a1
b1
c1
d1
SPAN 60M
SPAN 70M
SPAN 80M
SPAN
Cte < C
0.25
0.29
1.31

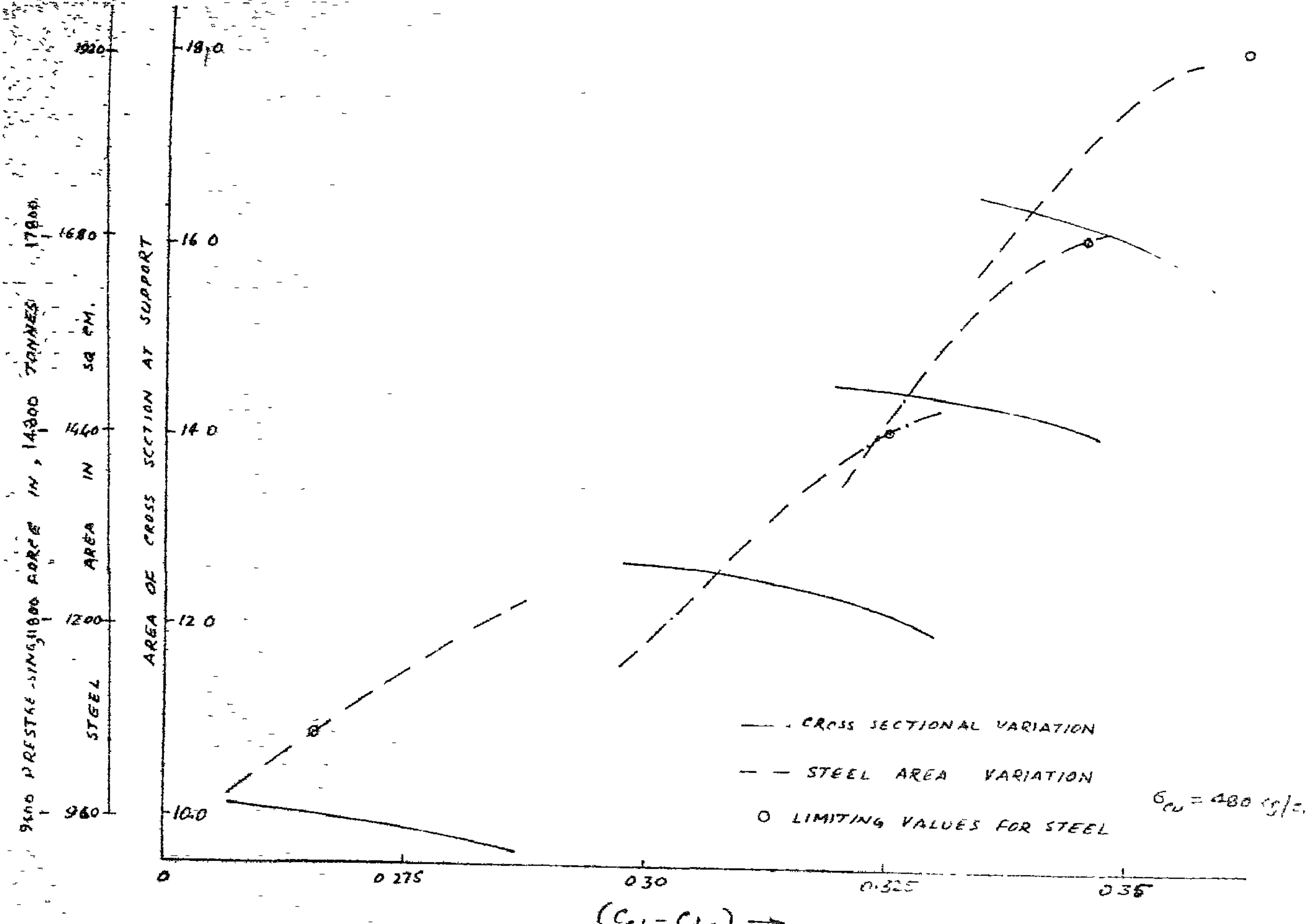


FIG. 3.15 – AREA OF

obtained by joining a_1, b_1, c_1 and d_1, gives the acceptable values of the area of cross section. Though a range of areas of cross section of design for every span is achieved, the lowest of the range cannot be adopted because of the constraint imposed on the amount of steel.

Fig. 3.14 is similar to Fig. 3.13 except that cube strength is 530 kg/cm^2 and curves A_5 A_5 and m_5 m_5 for 100 m span are included. Figs.3.13 and 3.14 give the amount of prestressing steel in terms of the steel ratio and the percentage of steel. It appears to be reasonable to have the value of steel ratio as 0.22 to 0.27 (or percentage of steel from 1.00 to 1.30) for spans from 60 to 100 m.

3.11. Design of Intermediate Sections:

After the sections at the support and free end are designed, the areas of cross section of the intermediate sections at the joints of the segments are calculated from the assumed variation of self weight. Suitable variations in sectional properties and the anchorage steel along the span are determined such that the stresses at these sections are within the permissible limits. A set of satisfactory variations is arrived at after a number of trials. The set of variations in sectional properties is found to ensure the working stresses below the permissible stresses. These values are modified in the program if necessary. In these trials, Eqs. 2.55 to 2.58 are used in the Eqs. 2.5 to 2.8 of permissible stresses.

a. The variation in depth of the section was assumed quadratic. This could not satisfy the stress constraints. Linear variation of the depth was found satisfactory. Hence it is suggested that the depth of the section be varied linearly from L/30 at some distance (d*) from free end to L/10 at the support. The depth of the section for a distance d* from the free end is constant at L/30, in order to ensure that the lower flange thickness is atleast equal to the minimum required. d* is found by trial.

b. A higher degree variation in the steel ratio yields a lesser length of prestressing steel. The variation in the steel ratio was first assumed cubic, but the resulting stresses were beyond permissible limits. The quadratic variation was found satisfactory. It is suggested that a value of 0.16 be adopted at the free end for the steel ratio ($\bar{m}_1$) and it be increased quadratically to the computed value at the support. Normally, the steel ratio required at the support is between 0.20 to 0.27.

c. The net eccentricity ($\bar{e}_1$) of the total prestressing force ($\bar{P}_{ti}$) at any section should increase with the distance of the section from the free end. The distance of centroid of the anchorage steel P_1, from the top fibre at any section is varied linearly from 0.55 times the depth at the free end to 0.30 times the depth at the section previous to the support.

d. Of all the stresses, the compressive stress at working load (c_{ce}) predominates at the sections near the support. One of the means to overcome this is to shift down the neutral axis. This is achieved by increasing the bottom flange thickness and decreasing the top flange thickness so that the cross sectional area is constant. It is therefore suggested that the top flange thickness be varied in the third degree from the free end.

3.12. Design Example:

An illustrative example of a cantilever of span 90.0 m is presented based on the program outlined in Chapter II.

1, Given:

The shape of the cross section chosen for the design is given in Fig. 2.10, with a road width of 7.5 m and footpath of 1.8 m on either side of the section.

span = 90 m

cube strength of the concrete = 530.0 kg./cm^2

ultimate tensile strength of prestressing steel = 14000.0 kg/cm^2

Loads:

The live and superimposed loads considered for the example are explained in Sec. 2.6.

b. Assumptions:

The area of cross section is assumed to vary quadratically along the span. The depth of the beam at free end is chosen as L/30 so as to meet the practical restraints of the flange thickness etc. (Sec. 2.6). The depth of beam at support is selected as 9.0 m (= L/10). The assumed value of η is 0.82. The centroid of prestressing steel is assumed to be at 0.06 h (54 cm.) from the top.

c. Design of the cross section at support:

(i) Area of cross section at free end:

Area of cross section at free end is calculated on the assumption given in Sec. 2.6.e.

thickness at bottom (assumed) = 0.16 m

thickness at top = $b_t/100 + 0.15$ (metres)

= 0.249 m

thickness of web = 2 (0.10 + depth at support/40.0)

= 0.65 m

depth of the beam L/30 = 3.0 m

area of cross section at free end = 5.101 m^2

(ii) Calculation of bending moments due to live and dead loads:

The bending moments are computed using a subroutine in the computer program.

bending moment due to superimposed D.L = 1.758×10^7 kg.m
bending moment due to L.L on footpath = 0.258×10^7 kg.m
bending moment due to L.L on bridge = 1.357×10^7 kg.m
(Max. due to class A)
moment due to external loads (M_a) = 3.373×10^7 kg.m

(iii) Calculation of the moment ratio (R) :

The compressive stress at working load is kept to the maximum permissible i.e. $c_{cc} = c'_{ce} = 0.28$. The other three coefficients are adjusted so as to satisfy the necessary constraints,

(a) to accommodate the steel within the section
(b) top flange thickness should not be less than minimum thickness required.

The calculations for the value of R are carried out till the differences between

(a) calculated and assumed value of α
(b) Δ calculated by the stress coefficients and Δ of the section
(c) the actual value of ρ and the value of ρ of the section

are within 3 percent of the assumed values.

The final values obtained from the computer program are:

$\Delta = 0.6511$ $\rho = 0.142$

$\alpha = 4.612$ $m = 0.2446$

$R = 0.4886$

The corresponding stress coefficients are,

$c_{tt} = -0.141$ (comp.) $c_{ct} = 0.4036$

$c_{tc} = -0.0787$ (comp.) $c_{cc} = c'_{cc} -$ [illegible] $= 0.280$

bending moment due to self weight (M_g) $= M$[illegible]/

$= 6.91 \times 10^7$ kg.m

area of cross section at support $= M_g/\alpha \gamma L^2$

$= 15.92\ m^2$

total prestressing force $= m A \sigma_{cu}$

$= 20090$ tonnes

(iv) Check for ultimate load:

The upper limit of the value of steel ratio of the calculated section as given by IS 1343-1960 $= 0.2765$

steel ratio provided $= 0.2448$

$m < m_{limit}$

Taking the live and dead load factors (IS 1343-1960) as 3.0

and 1.5 respectively,

the ultimate bending moment required $= 17.997 \times 10^7$ kg.m

ultimate moment capacity of the section $= 22.905 \times 10^7$ kg.m

M_u of beam $>$ M_u required

The design of the section at support conforms to the code limitations.

c. Design of the intermediate sections:

The longitudinal variation in section and steel ratios are found out on the basis of contents of Sec.3.10 so as to ensure the working stresses of all the sections within the permissible limits. The sectional details, prestressing force and steel area are given in Fig. 3.16 and Table 3.1.

The total volume of concrete (excluding for footpath) in the above example is equal to 783.9 m^3. The volume of concrete needed with the same concrete strength and steel ratio is 1078.4 m^3 when the cross section assumed to vary linearly.

TABLE 3.1: SECTIONAL DETAILS OF FIG. 3.16 = (t_w=0.65, c_f=b_b/b_t = 0.60)

Section No. i 1.	x (m) 2.	depth (m) 3.	t_t (m) 4.	t_b (m) 5.	A (m^2) 6.	Area of steel introduced A_{si} (cm^2) 7.	P_{ti} (Tons) 8.	Distance of P_{ti} from top (m) 9.	$\bar{m}_i$ 10.	ϵ_i 11.
1	0.0	3.0	0.248	0.16	5.101	444.1	4352.6	1.50	0.1600	-0.1285
2	3.913	3.0	0.248	0.164	5.120	2.27	22.2	1.47	0.1602	0.0220
3	7.826	3.0	0.248	0.176	5.182	6.83	66.9	1.41	0.1607	0.1734
4	11.739	3.285	0.249	0.160	5.284	8.58	81.1	1.55	0.1616	0.3136
5	15.652	3.571	0.249	0.152	5.427	16.22	159.0	1.66	0.1628	0.3080
6	19.565	3.857	0.249	0.151	5.611	21.14	207.2	1.76	0.1644	0.3080
7	23.478	4.143	0.249	0.158	5.836	26.28	257.6	1.85	0.1665	0.3100
8	27.391	4.428	0.249	0.173	6.102	31.65	310.2	1.94	0.1686	0.3190
9	31.304	4.714	0.249	0.196	6.409	37.34	365.9	2.03	0.1712	0.3304
10	35.217	5.000	0.249	0.227	6.757	43.38	425.1	2.10	0.1742	0.3443
11	39.130	5.286	0.250	0.265	7.146	49.82	488.2	2.18	0.1775	0.3600
12.	43.044	5.571	0.250	0.310	7.575	56.71	557.4	2.25	0.1812	0.3769
13.	46.957	5.857	0.251	0.363	8.046	64.08	628.0	2.31	0.1852	0.3944

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
14.	50.869	6.143	0.252	0.423	8.557	71.99	705.5	2.37	0.1896	0.4122
15.	54.783	6.428	0.253	0.490	9.109	80.46	788.7	2.43	0.1943	0.4297
16.	58.696	6.714	0.255	0.563	9.705	89.61	878.1	2.48	0.1994	0.4467
17.	62.609	7.00	0.257	0.645	10.337	42.56	415.3	2.52	0.1948	0.4776
18.	66.522	7.28	0.260	0.731	11.0121	81.88	802.4	2.56	0.1966	0.4926
19.	70.434	7.57	0.264	0.826	11.728	89.76	879.6	2.60	0.1987	0.5050
20.	74.348	7.86	0.268	0.926	12.485	85.52	838.1	2.63	0.1992	0.5174
21.	78.261	8.14	0.274	1.031	13.283	150.95	1479.3	2.65	0.2000	0.5199
22.	82.174	8.42	0.261	1.142	14.121	254.89	2301.9	2.67	0.2252	0.5165
23.	86.087	8.71	0.269	1.259	15.002	313.9	3077.1	2.69	0.2147	0.5235
24.	90.00	9.0	0.30	1.321	15.92	0	–	–	0.258	0.527

CHAPTER IV

CONCLUSIONS

The effect and behaviour of various parameters were studied and the following conclusions are arrived at. Certain suggestions in approach to the problem and design recommendations are made. The number of trials for a designer is made least when the range of values for various parameters are chosen as given below.

4.1. General:

a. Considering the present practice in bridge design with respect to L.L/D.L effects, depth/span ratio and the stresses allowable in the code, it has been found that the upper limit of span in the case of simply supported case is about 140 m. In the case of cantilever bridges, the limit is about 212 m (pier spacing). The above span limits are found to be sensitive to the shape of the section, the span/depth ratio and the strength of concrete.

b. An attempt has been made to develop a design procedure for fully stressed design of the most critical section in cantilever, namely the support section. It was found that a fully stressed design is almost impossible because of the physical constraints. Most of the actual stresses in the section are calculated as close as possible to the full permissible stresses so that the minimum practical constraints are satisfied. It is always possible to permit the maximum allowable stress in compression at working load condition.

c. The quadratic variation of the increase in the area of cross section along the length of the beam leads to a weight less than the one with linear variation. Any higher order variation of the cross section is leading to some unacceptable intermediate section. Even the quadratic variation of cross section raises some difficult acceptable intermediate sections. However, it is possible to arrive at the acceptable sections at all intermediate points by a particular variation in the thickness of the top and bottom flanges. An acceptable section is the one which satisfies all the permissible stress criteria while conforming to practical and minimum requirements.

4.2. Design Parameters:

a. The shape of the cross section of a bridge fixes the working stress. It is important to choose a correct shape factor. The value of shape factor is between 1.15 and 0.60 for spans from 40 m to 100 m.

b. It is desirable to have higher strength concrete as the span increases. The minimum weight is obtained by adopting maximum depth at support.

c. It is desirable to have a maximum efficiency ratio of the section. It appears to be reasonable to have the value of efficiency ratio in the range of 0.135 to 0.148 for spans from 100 to 40 m.

d. Reasonable values of self weight bending coefficient can be given in terms of span as follows.

(i) for area of cross section varying linearly (upto 80.0 m)

$$\alpha = 2.940 + 0.025\ (L - 40.0)$$

(ii) for area of cross section varying quadratically (upto 100 m)

$$\alpha = 3.125 + 0.029\ (L - 40.0)$$

e. The maximum value of steel ratio is approximated as 0.22 to 0.27 for spans from 60 m to 100 m.

f. Value of moment ratio increases from 0.45 to 0.80 in an economic design as the span decreases from 100 to 40 m.

4.3. Recommendations:

The following tentative recommendations are made.

a. The free end section is designed subject to practical limits. Whereas the area of cross section at the support may be designed using the following guidelines.

b. The rate of increase in the area of cross section along the span shall be taken to be quadratic.

c. The rate of increase in the depth of section is linear.

d. The rate of increase of percentage of steel is quadratic.

e. Position of anchorage steel from top edge decreases linearly from the free end towards the support.

f. A check for stresses at intermediate points is required.

g. A high strength concrete consistent with economy be used.

h. A thinner web consistent with safety against buckling and higher ratio of width of bottom to top flange generally lead to an economical design.

i. The box section has an inherent strength against torsion and lateral stability. Therefore the design is not usually governed by these two criteria.

j. The prestressing force which produces compressive stress raises the level of shear capacity of the section. In addition, the vertical component of the cable force balances some portion of the external shear force. Hence the design for shear is of secondory nature.

k. Check for deflections at transfer and working load conditions is necessary.

4.4. Future Work:

(i) The analysis of the structure by 'Load Balancing Method' is thought to be an effective method since the major part of the total load of the structure is known, namely the dead load. The method of load balancing is suggested for future work.

(ii) Cost of handling, launching, etc. are considerable, therefore minimising the total ~~weight~~ Cost of the structure is very desirable. An optimisation program with the necessary physical constraints will be an improvement over the study.

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APPENDIX I

COMPUTER PROGRAM

A brief description of each of the Fortran subroutines is given in Section I-1. A flow diagram of the main program is included in I-2.

I-1. Description of Subroutines:

1. Subroutine stress:

'Subroutine **STRESS**' computes the stresses both at transfer and working conditions from the four governing equations. The input data for subroutine stress are delta, ρ , eccentricity of steel, steel ratio, alpha, η and R.

2. Subroutine COEF:

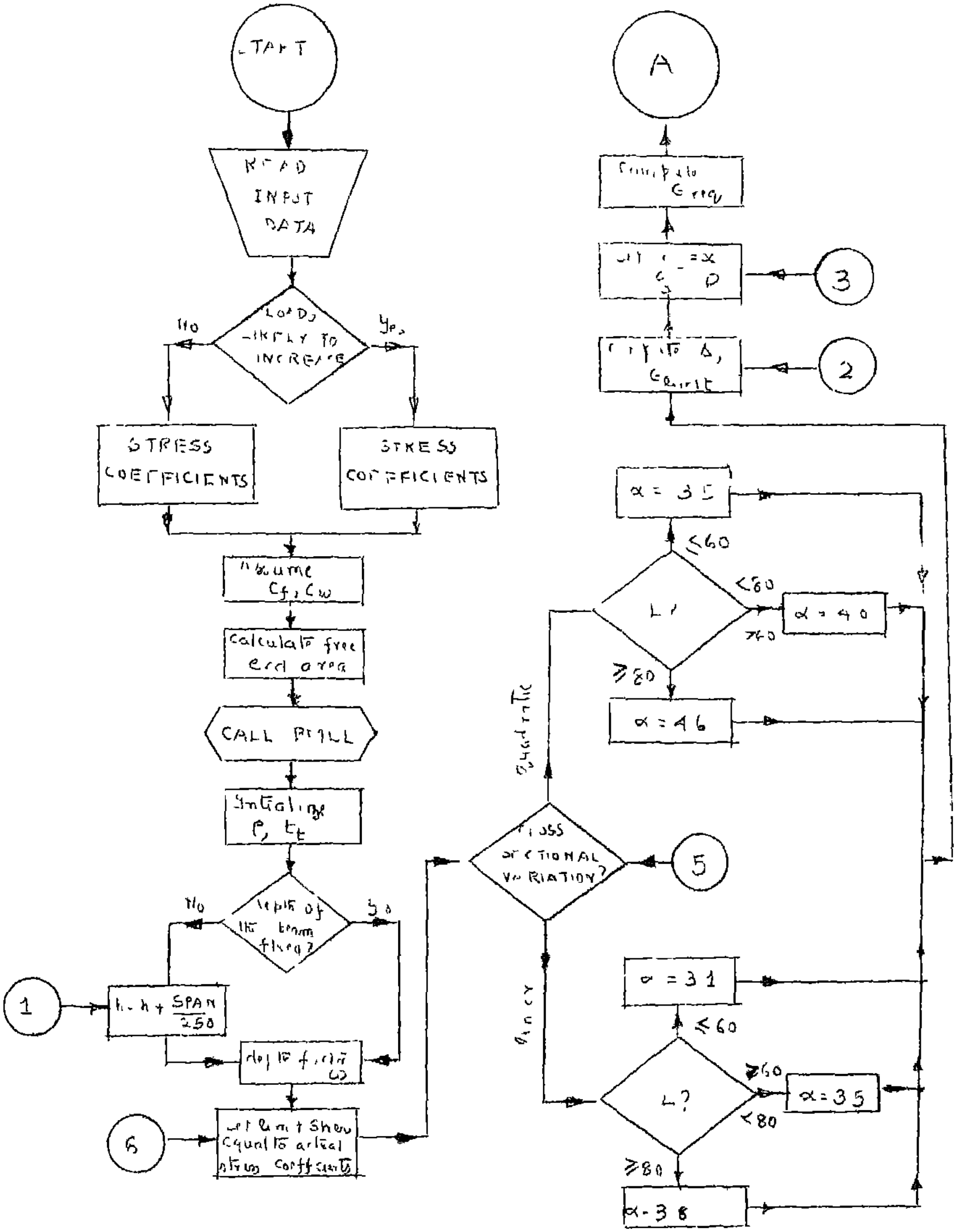
'Subroutine COEF' gives the sectional properties ρ, c_{yt}, c_{yb}, and from the non-dimensionalized factors c_t, c_b, c_f and c_w.

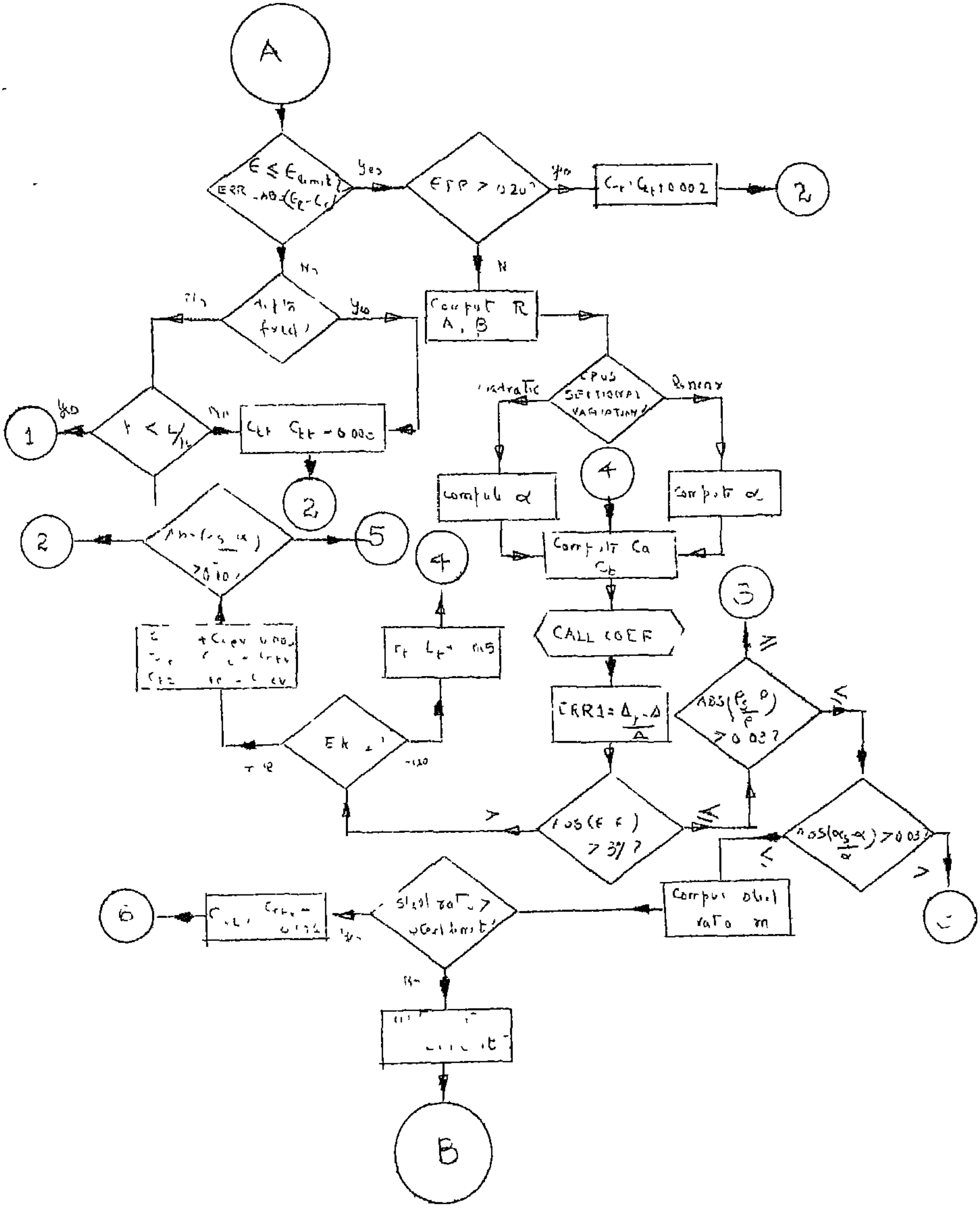
3. Subroutine OM:

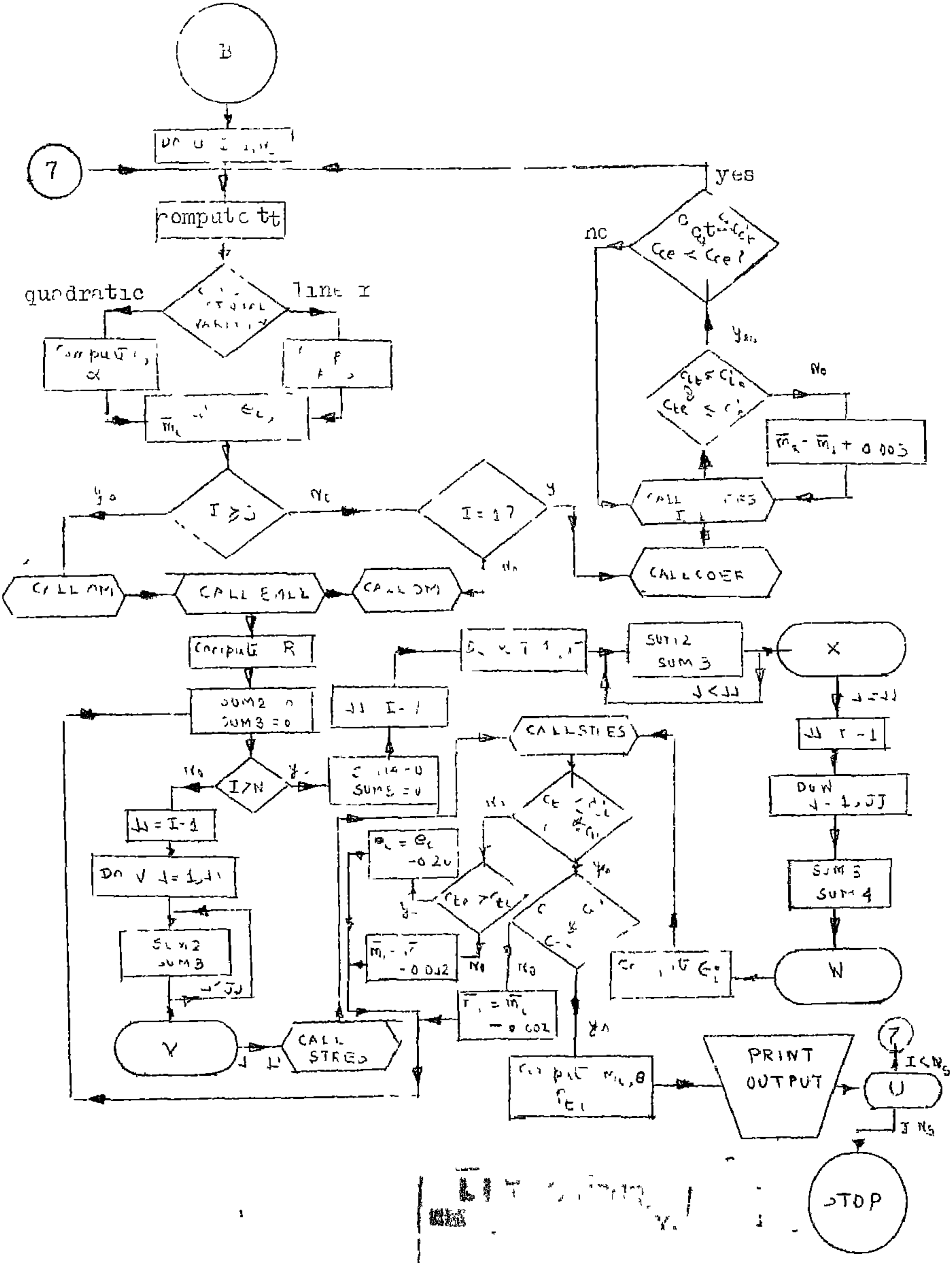
This routine computes the two coefficients c_a, c_b and calls the subroutine coef. The depth factor is also calculated in 'subroutine OM'.

4. Subroutine BMLL:

'Subroutine BMLL' considers all the possible cases of


START
READ INPUT DATA
LOADS LIKELY TO INCREASE
No
Yes
STRESS COEFFICIENTS
STRESS COEFFICIENTS
Assume C_f, C_w
Calculate free end area
CALL PMILL
Initialize P, t_t
Depth of the beam fixed?
No
Yes
1
h = h + SPAN/250
depth fixed
6
set limit stress equal to actual stress coefficients
CROSS SECTIONAL VARIATION?
5
Quadratic
Linear
L?
≤60
<60
≥40
≥80
α = 35
α = 40
α = 46
L?
≤60
≥60
<80
≥80
α = 31
α = 35
α - 38
Compute A, G_{crit}
2
set $C_g = \alpha P$
3
Compute G_{req}
A


A
E ≤ E limit?
ERR -ABS(Eₜ-Cₜ)
Yes
ERR > 0.20?
Yes
N
No
Yes
No
Compute R A, B
Yes
No
1
2
2
5
4
4
3
CROSS SECTIONAL VARIATION?
Quadratic
Linear
Compute α
Compute α
Compute Ca Cb
CALL COEF
ERR1 = Δ - Δ / Δ
ABS(ρs - ρ / ρ) > 0.03?
≥
≤
ABS(E F) > 3%?
>
≤
ABS(αs - α / α) > 0.03?
<
>
Ek
Compute slab ratio m
Yes
No
6
B


B
7
yes
compute tt
nc
quadratic
line r
no
I ≥ J
I = 1 ?
CALL COER
CALL ENLL
CALL DM
$\overline{m}_i = \overline{m}_i + 0.005$
Compute R
SUM2 = 0
SUM3 = 0
I > N
JJ = I - 1
DO V J = 1, JJ
SUM2
SUM3
J < JJ
V
CALL STRES
CALL STRES
SUM2
SUM3
X
JJ = I - 1
DO W J = 1, JJ
SUM3
SUM4
W
$\overline{m}_i = \overline{m}_i - 0.002$
$\theta_i = \theta_i - 0.20$
PRINT
OUTPUT
7
I < Ns
U
I = Ns
STOP

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Date Slip

This book is to be returned on the date last stamped

Thesis
624.35
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Swami,
Minimum weight design of cantilever bridges.